ABSTRACT BOOKLET

WELCOME

We would like to welcome you to IWOBIP'18, the 2nd International Workshop on Bilevel Programming. After the first edition in 2016 held in Monterrey, Mexico in 2016, IWOBIP'18 will take place at INRIA Lille-Nord Europe, France from June 18 to 22, 2018.

Like the previous edition, IWOBIP'18 aims to provide a forum for scientific exchange and cooperation in the field of bilevel programming and related areas. IWOBIP'18 will start by lectures on Monday and Tuesday morning given by Didier Aussel and Patrice Marcotte. The workshop will start on Tuesday afternoon. This year we have Tobias Harks, Ivana Ljubic and Miland Tambe as plenary speakers and a set of 52 presentations.

We would like to thank Nathalie Bonte, Brigitte Foncez, Corinne Jamroz and Christine Yvoz for their administrative support.

We would also like to thank Centrale Lille, CNRS, CRIStAL (Centre de Recherche en Informatique Signal et Automatique de Lille), INRIA, PGMO (Programme Gaspard Monge – Fondation Jacque Hadamard), ROADEF (the French Operations Research Society) and the Région des Hauts-de-France for being co-sponsors of this event.

We wish all participants a very fruitful conference.

Luce Brotcorne and Martine Labbé

Plenary speaker : **Milind Tambe**

Helen N. and Emmett H. Jones Professor in Engineering
University of Southern California

Multiagent Systems Research for Social Goods: The Role of Bilevel Programming

With the maturing of AI and multiagent systems research, we have a tremendous opportunity to direct these advances towards addressing complex societal problems. One key multiagent systems challenge that cuts across multiple of these problem areas is that of effectively deploying limited intervention resources. I will highlight our research advances rooted in computational game theory in addressing this challenge across three key problem areas; and in particular highlight the role of bilevel programming to address Stackelberg game models. First, I will focus on public safety and security, and outline our contribution in introducing and using the Stackelberg security games model for effectively allocating limited security resources. Security games models been used by agencies such as the US Coast Guard, the US Federal Air Marshals Service and others to assist in the protection of ports, airports, flights and other critical infrastructure. Second, I will focus on conservation and illustrate the use of green security games to allocate limited resources in protecting endangered wildlife. Advances in adversary modeling in these games -- models learned from past poaching data -- have helped removal of snares and arrests of poachers in national parks in Uganda, potentially saving endangered animals. Third, for public health, I will outline challenges of using limited resources for spreading health information in low resource communities, and algorithms based on games against nature. Our new algorithms for influence maximization, piloted in homeless shelters in Los Angeles, show significant improvements over traditional methods in harnessing social networks to spread HIV-related information among homeless youth. I will also point to directions for future work, illustrating the significant potential of AI for social good.

Plenary speaker : **Ivana Ljubic**

ESSEC Business School, Paris

New Branch-and-Cut Algorithms for Mixed-Integer Bilevel Linear Programs

In this talk we focus on new branch-and-cut (B&C) algorithms for dealing with mixed-integer bilevel linear programs (MIBLPs). MIBLPs constitute a significant family of bilevel optimization problems, where all objective functions and constraints are linear, and some/all variables are required to take integer values. We first consider a general case in which the proposed B&C scheme relies on intersection cuts derived from feasible-free convex sets, see [1,2]. Our B&C scheme is finitely-convergent in case both the leader and the follower problems are pure integer. In addition, it is capable of dealing with continuous variables both in the leader and in follower problems—provided that the leader variables influencing follower's decisions are integer and bounded. Our new algorithm consistently outperforms (often by a large margin) all alternative state-of-the-art methods from the literature, including methods which exploit problem specific information for special instance classes. In particular, it allows to solve to optimality more than 300 previously unsolved instances from literature. We then focus on a subfamily of MIBLPs in which the leader and follower typically share a set of items, and the leader can select some items and interdict their usage by the follower. The only constraints in the follower subproblem involving leader decision variables impose that, if an interdiction variable is selected by the leader, then certain actions of the follower are inhibited. Interdiction Problems, Blocker Problems, Critical Node/Edge Detection Problems are some examples satisfying the later condition. We show that, in case the follower subproblem satisfies monotonicity property, a family of "interdiction-cuts" can be derived resulting in a more efficient B&C scheme. Computational results are reported for the (multidimensional) knapsack interdiction and the maximum clique interdiction problems, see [3,4].

Literature:

[1] M. Fischetti, I. Ljubic, M. Monaci, M. Sinnl, On the Use of Intersection Cuts for Bilevel Optimization, Mathematical Programming, online first, 2017, DOI 10.1007/s10107-017-1189-5

[2] M. Fischetti, I. Ljubic, M. Monaci, M. Sinnl, A new general-purpose algorithm for mixed-integer bilevel linear programs, Operations Research 65(6): 1615-1637, 2017

[3] M. Fischetti, I. Ljubic, M. Monaci, M. Sinnl: Interdiction Games and Monotonicity, with Application to Knapsack Problems, INFORMS Journal on Computing, to appear, 2018

[4] F. Furini, I. Ljubic. P. San Segundo, S. Martin: The Maximum Clique Interdiction Game, submitted, 2018

Plenary speaker : **Tobias Harks**

Institute of Mathematics
University of Augsburg.

The Continuous Bilevel Network Design Problem

I will talk about a classical problem in transportation, known as the (bilevel) continuous network design problem, CNDP for short. Given a graph for which the latency of each edge depends on the ratio of the edge flow and the capacity installed, the goal is to find an optimal investment in edge capacities so as to minimize the sum of the routing costs of an induced Wardrop equilibrium and the investment costs for installing the edge's capacities. In this talk, I will discuss several classical algorithms for computing provably good solutions together with a complexity-theoretic classification of the problem.

Consistent Conjectures Coincide With Optimal Nash Strategies in the Upper Level Game

Viacheslav Kalashnikov*
School of Engineering and Sciences (EIC), Tecnológico de Monterrey (ITESM); and Department of Experimental Economics, Central Economics and Mathematics Institute (CEMI), Moscow, Russia; and Department of Computer Science, Sumy State University, Sumy, Ukraine, `kalash@itesm.mx`

José G. Flores-Muñiz
Department of Physics and Mathematics, Universidad Autónoma de Nuevo León (UANL), `slavkamx@gmail.com`

Nataliya Kalashnykova
Department of Physics and Mathematics, Universidad Autónoma de Nuevo León (UANL), `nkalash2009@gmail.com`

Abstract Conjectural variations equilibrium (CVE) was introduced quite long ago as another possible solution concept in static games. According to this concept, agents behave as follows: each agent chooses his/her most favorable action taking into account that every rival's strategy is a *conjectured function* of his/her own strategy.

In [1], a new concept of conjectural variations equilibrium (CVE) was introduced and investigated, in which the conjectural variations (represented via the influence coefficients of each agent) affected the structure of the Nash equilibrium.

The detailed story of the highs and lows of the CVE concept is described in [2]. The main obstacle in the way of admitting this concept is its consistency. The consistency (or, sometimes, "rationality") of the equilibrium is defined as the coincidence between the conjectural best response of each agent and the conjectured reaction function of the same.

To cope with a conceptual difficulty arising in many player models, a completely new approach was proposed as applied later to mixed oligopoly models [2]. Consider a group of n producers ($n \geq 3$) of a homogeneous good with the cost functions $f_i(q_i)$, $i = 1, \ldots, n$, where $q_i \geq 0$ is the output by producer i. Consumers' demand is described by a demand function $G = G(p)$, whose argument p is the market clearing price. Active demand D is non-negative and does not depend upon the price. The equilibrium between the demand and supply for a given price p is guaranteed by the following balance equality

$$\sum_{i=1}^{n} q_i = G(p) + D. \tag{1}$$

Every producer $i = 1, \ldots, n$, chooses her output volume $q_i \geq 0$ so as to maximize her profit function $\pi_i(p, q_i) = p \cdot q_i - f_i(q_i)$. Now we postulate that the agents (producers) suppose that their choice of production volumes may affect the price value p. The latter assumption could

be defined by a conjectured dependence of the price p upon the output values q_i. If so, the first order maximum condition to describe the equilibrium would have the form:

$$\frac{\partial \pi_i}{\partial q_i} = p + q_i \cdot \frac{\partial p}{\partial q_i} - f_i'(q_i) \begin{cases} = 0, & \text{if } q_i > 0; \\ \le 0, & \text{if } q_i = 0, \end{cases} \quad \text{for} \quad i = 1, \ldots, n. \tag{2}$$

Thus, we see that to describe the agent's behavior, we need to evaluate the behavior of the derivative $\partial p / \partial q_i = -v_i$ rather than the functional dependence of p upon q_i. Then the optimality condition (2) is reduced to

$$\begin{cases} p = v_i q_i + b_i + a_i q_i, & \text{if } q_i > 0; \\ p \le b_i, & \text{if } q_i = 0. \end{cases} \tag{3}$$

Definition 1. A collection $(p, q_1, \ldots, q_n)$ is called an *exterior equilibrium state* for given influence coefficients $(v_1, \ldots, v_n)$, if the market is balanced, i.e., equality (1) holds, and for each i the maximum conditions (3) are valid.

We assume the following properties of the model's data.

A1. The demand function $G = G(p) \ge 0$ is defined for $p \in (0, +\infty)$, being non-increasing and continuously differentiable.

A2. For each i, the cost function f_i is quadratic and $f_i(0) = 0$, i.e.,

$$f_i(q_i) = (1/2) a_i q_i^2 + b_i q_i, \quad \text{where} \quad a_i > 0, b_i \ge 0,\ i = 1, \ldots, n. \tag{4}$$

A3. For the price value $p_0 = \max_{1 \le j \le n} b_j$, the following estimate holds:

$$\sum_{i=1}^{n} \frac{p_0 - b_i}{a_i} < G(p_0). \tag{5}$$

Now one can prove the following

Theorem 1. *Under assumptions A1, A2, and A3, for any $D \ge 0$, $v_i \ge 0$, $i = 1, \ldots, n$, there exists uniquely the exterior equilibrium $(p, q_1, \ldots, q_n)$ depending continuously upon the parameters $(D, v_1, \ldots, v_n)$. The equilibirum price $p = p(D, v_1, \ldots, v_n)$ as a function of these parameters is differentiable with respect to D and v_i, $i = 1, \ldots, n$. Moreover, $p(D, v_1, \ldots, v_n) > p_0$, and*

$$\frac{\partial p}{\partial D} = \frac{1}{\sum_{i=1}^{n} \frac{1}{v_i + a_i} - G'(p)}. \tag{6}$$

Now having formula (6) in mind and following the ideas of [2]-[3], we introduce the

Consistency Criterion. At the exterior equilibrium $(p, q_1, \ldots, q_n)$, the influence coefficients v_k, $k = 1, \ldots, n$, are referred to as *consistent* if the equalities below hold:

$$v_k = \frac{1}{\sum_{i=1, i \ne k}^{n} \frac{1}{v_i + a_i} - G'(p)}, \quad k = 1, \ldots, n. \tag{7}$$

Now we are in a position to define the concept of interior equilibrium.

Definition 2. A collection $(p, q_1, \ldots, q_n, v_1, \ldots, v_n)$ is called an *interior equilibrium state*, if for the considered influence coefficients, the collection $(p, q_1, \ldots, q_n)$ is the exterior equilibrium, and the consistency criterion is valid for all $k = 1, \ldots, n$. The interior equilibrium existence result is as follows:

Theorem 2. *Under assumptions A1, A2, and A3, there exists an interior equilibrium state.*

Theorem 1 allows us to define the following game $\Gamma = (N, V, \Pi, D)$, which will be called the *upper level game*. Here, D is a (fixed) value of the active demand, $N = \{1, \ldots, n\}$ is the set of the same players as in the above-described model, $V = R_+^n$ represents the set of possible strategies, i.e., vectors of conjectures $v = (v_1, \ldots, v_n) \in R_+^n$ accepted by the players, and finally, $\Pi = \Pi(v) = (\pi_1, \ldots, \pi_n)$ is the collection of the payoff values defined (uniquely by Theorem 1) for the strategy vector v.

Now the main results of this talk are as follows. In general, the Cournot conjectures are not consistent in our single commodity market model. In other words, the Cournot conjectures $v_i = -p'(G)$ do not usually satisfy the (nonlinear) consistency system (7). However, in the upper level game introduced above, the consistent conjectures, determined by (7) provide the Nash equilibrium. The theorem below was established in [3]:

Theorem 3. *The consistent influence coefficients $v_k,\ k = 1, \ldots, n$, determined as the unique (for a fixed p) solution of system (7) form a vector of the Nash equilibrium in the upper level game.*

Since the upper-level-game strategies set $V = \mathbb{R}_+^n$ is unbounded, the existence of at least one Cournot-Nash equilibrium state in this game is by no means easy to check. The following two results (under some extra assumptions) guarantee that the existence of interior equilibrium in the original oligopoly implies the existence of Nash equilibrium in the upper level.

Theorem 4. *If, in addition to assumptions A1–A3, the demand function G is concave, then the consistency criterion for the original oligopoly is a necessary and sufficient condition for the collection of conjectures $v = (v_1, \ldots, v_n)$ to provide a Cournot-Nash equilibrium state in the upper level game.*

As the concavity of the demand function might be a too restrictive condition, the next theorem relaxes it by replacing the latter with the Lipschitz-continuity of the derivative $G'(p)$.

Theorem 5. *Suppose that apart from assumptions A1–A3, the demand function's derivative is Lipschitz- continuous. In more detail, for assume that for any $p_1 > 0$ and $p_2 > 0$ the following inequality holds:*

$$|G'(p_1) - G'(p_2)| \leq \frac{1}{2s^2 G(p_0)} |p_1 - p_2|, \tag{8}$$

where $s = \max\{a_1, \ldots, a_n\}$, and the price value p_0 is defined in assumption A3. Then, the consistency criterion for the original oligopoly is a necessary and sufficient condition for the collection of influence conjectures $v = (v_1, \ldots, v_n)$ to be a Cournot-Nash equilibrium in the upper level game.

Keywords: *Consistency, Conjectural variations equilibrium, Upper level game*

References

[1] Isac, G., Bulavsky, V.A., and V.V. Kalashnikov, "Complementarity, Equilibrium, Efficiency and Economics," Kluwer Academic Publishers, Dordrecht, the Netherlands, 2002.

[2] Kalashnikov, V.V., Bulavsky, V.A., Kalashnykova, N.I., and F.J. Castillo, *Mixed oligopoly with consistent conjectures*, European J. Oper. Res. **201** (2011), 729–735.

[3] Kalashnikov, V.V., Bulavsky, V.A., Kalashnykova, N.I., and F. López-Ramos, *Consistent conjectures are optimal Cournot-Nash strategies in the meta-game*, Optimization, **66** (2017), 2007–2024.

Constraint qualifications for parametrized optimization problems and applications to multi-leader-follower games

Anton Svensson*
CMM , Universidad de Chile, `asvensson@dim.uchile.cl`

Didier Aussel
Lab PROMES, Université de Perpignan, `aussel@univ-perp.fr`

Abstract Parametric optimization problems are very important in applications, for instance, to define noncoopertive games such as Nash games or Bilevel programs, which are particular cases of the class of multi-leader-follower games. To ensure that a parametrized convex optimization problem (convex for each parameter) is equivalent to its parametrized KKT conditions, one could verify that a constraint qualification is satisfied for each parameter. We show a simple way for doing so by assuming joint convexity of the parametrized optimization problem, that is, the functions that define the constraints are convex on the joint vector composed by a solution and a parameter. We show how this applied for comparing solutions of a Generalized Nash Equilibrium Problem (GNEP) with the concatenation of KKT conditions of all players, and the solutions of a multi-leader-follower game and its MPCC reformulation, obtained by replacing the lower level GNEP by its concatenated KKT conditions.

Keywords: *Constraint qualification, Parametric optimization, Joint convexity, Multi-leader-follower games, Optimality conditions*

References

[1] Dempe, S., and J. Dutta, *Is bilevel programming a special case of mathematical program with complementarity constrinats?*, Math. Prog. **9** (2012), 37–48.

[2] Ehrenmann, A. , Equilibrium problems with equilibrium constraints with their application to electricity markets, PhD thesis, 2004.

Contributed session

Analysis of a Game of Dice: Bilevel Optimization Problem under Uncertainty Joining a Markov Decision Process

Johanna Burtscheidt*
Faculty of Mathematics, University of Duisburg-Essen, johanna.burtscheidt@uni-due.de

Alexander Frank
Department of Computer Science, University of Dortmund, alexander.frank@cs.tu-dortmund.de

Abstract The analysis of stochastic games is an important part of game theory. In such games, separate rounds are frequently uncorrelated and so they can be described as Markov chain or Markov Decision Process (MDP). As an example of such stochastic games, we studied a special game of dice with three dice for two (or more) players. Here each participant has to choose one of three strategies in every round to maximize its score. In doing so they have to make their decision without knowing the realization of some randomness, as among other things the outcome of two dice is unknown. This type of problem can be formulated as a Bilevel Optimization Problem (BP) under stochastic uncertainty, in which both the leader and the follower have a lack of information.

We will construct several deterministic bilevel formulations which allow to take risk aversion into account. For a selection of 'good' strategies, some similar instances of the dice game will be juxtaposed. In addition to that, we compare performance and complexity of our modells to ordinary MDP formulations of the given problem. The objective is to compare two known mathematical methods for a stochastic decision game on different point of views.

Keywords: *Markov Decision Process, Bilevel Optimization Problem, Risk Aversion, Uncertainty*

About Strong Stackelberg Equilibrium in Stochastic Games

Víctor Bucarey L.*
Departamento de Ingeniería Industrial, Universidad de Chile , vbucarey@ing.uchile.cl

Eugenio Della Vecchia
Departmento de Matemática , Universidad Nacional de Rosario,
eugenio@fceia.unr.edu.ar

Alain Jean-Marie
INRIA - Sophia Antipolis Meditarreneé, alain.jean-marie@inria.fr

Fernando Ordoñez
Departamento de Ingeniería Industrial, Universidad de Chile , fordon@dii.uchile.cl

Abstract In this work we face the problem of computing a strong Stackelberg equilibrium (SSE) in a stochastic game (SG). Given a set of states we model a two player perfect information dynamic where one of them, called *Leader* or player A, observes the current state and decides, possible up to probability distribution f, between a set of available actions. Then other player, called *Follower* or player B, observes the strategy of player A and plays his best response noted by g. We represent a two-person stochastic discrete game $\mathcal{G}$ by

$$\mathcal{G} = (\mathcal{S}, \mathcal{A}, \mathcal{B}, Q, r_A, r_B, \beta_A, \beta_B, \tau) \ .$$

where $\mathcal{S}$ represents the states of the games, $\mathcal{A}$ and $\mathcal{B}$ represents space of actions of both players. We denote $\mathcal{A}_s$ and $\mathcal{B}_s$ the available set of actions of both players in state s. $Q = Q^{ab}(z|s)$ represents the transition probability of going from a state s to a state z given the actions a and b were performed by players A and B respectively. $r_A^{ab}(s)$, $r_B^{ab}(s)$ represents the one-step rewards functions depending on the actual state, and the actions performed by the players. β_A and β_B are the discount factors for both players. τ represents the number of periods in the dynamic.

Stochastic games were first introduced by Shapley [1] who also gives the first algorithm to find Nash Equilibrium in zero-sum SG based on a dynamic programming algorithm. SG have been used to model interaction in economics, computer networks, and security, among others applications. Feedback policies are policies depending on the actual state s and time epoch t. Stationary policies can be defined as feedback policies that do not depend on the time step. In our setting, players aim to maximize their expected discounted sum of payoffs from 0 until τ considering feedback and stationary policies.

For the finite horizon setting, Value iteration can be used to find an Strong Stackelberg equilibrium in feedback policies [2]. In contrast to MDP settings, Stackelberg equilibrium in stationary policies can be arbitrary suboptimal as is showed in [3]. Authors also provide a

Mixed integer non linear program to compute a SSE in general SG when players are restricted to stationary policies. To the best of our knowledge there is no prior work on efficient algorithms to find stationary strategies in Stackelberg games for $\tau \to +\infty$ when they exist.

We show that for instances, where the best response of the follower does not depend on its future values, dynamic programmig based techniques compute the values and the Strong Stackelberg Strategies in stationary policies. To do so, we define suitable operators which are monotone and contractive. As a consequence of that we can also show that Policy Iteration converges to a Stackelberg Equilibrium in stationary policies.

We prove that for the general case it is no always possible to converge to a Stackelberg Equilibrium via a counterexample. Furthermore, in this counterexample we are able to compute multiple Stackelberg Equilibrium in stationary policies, so the question of computes SSE in SG in stationary policies remains open.

We discuss about mathematical programming models to compute Strong Stackelberg Equilibrium. Finally we test computationally a dynamic programming type algorithm to compute stationary policies forming equilibrium in Security Games.

Keywords: *Stochastic Games, Strong Stackelberg Equilibrium, Dynamic Programming*

References

[1] L. S. Shapley, *Stochastic games*, Proceedings of the national academy of sciences. **39** (1953), 1095–1100.

[2] L. S. Shapley, *Dynamic noncooperative game theory*, SIAM. **200** (1995), 1095–1100.

[3] Y. Vorobeychik and S. Singh, *Computing stackelberg equilibria in discounted stochastic games (corrected version)*, Proceedings of the national academy of sciences. (2012)

A Resource Allocation Model for Pandemic Planning and Control

David Rey*
School of Civil and Environmental Engineering, University of New South Wales (UNSW) Sydney, `d.rey@unsw.edu.au`

Aleksa Zlojutro
School of Civil and Environmental Engineering, University of New South Wales (UNSW) Sydney, `azlojutro@hotmail.com`

Lauren Gardner
School of Civil and Environmental Engineering, University of New South Wales (UNSW) Sydney, `l.gardner@unsw.edu.au`

Abstract

The modern world is highly connected via air travel, enabling infectious diseases to spread long distances in short periods of time. Epidemics can be mitigated by identifying and isolating infected travelers before they have the chance to enter into a new population. However, successful screening of infected passengers is expensive due to personnel and/or new technology costs, and can not feasibly be conducted on all passengers at all airports. This work addresses the decision of where to allocate available screening resources in the event of a novel emerging infectious disease epidemic. Copious studies have focused on either accurately modeling outbreak dynamics or optimizing outbreak control decisions. However, few efforts have attempted to bridge these two fields, leaving a gap in the literature for optimization-based outbreak control models that account for the heterogeneous nature of infectious disease spreading dynamics.

To address this gap in the literature we propose a bi-level programming formulation to represent this decision problem. The upper level problem is to identify a feasible resource allocation strategy which minimizes the impact of the outbreak and is subject to a pre-defined budget constraint; the lower level problem evaluates a control strategy using a stochastic dynamic outbreak simulation model. The work builds on previous work which first mathematically defined the problem framework [1] and integrated a deterministic version of the multi-scale simulation model [2].

In this work the lower level model is a stochastic dynamic simulation model designed to capture the disease spreading process that occurs due to human interactions at both the local (city) and global (air travel) scale. To model the evolution of an infectious disease within a city we use a deterministic compartmental model [3], specifically the SEIR model, where S represents the group of individuals susceptible to the disease, E represents the individuals who have been infected but are not yet contagious, I represents the individuals that have been infected and are contagious, and R represents the individuals that have been previously infected, but are now recovered (can not be infected again). The local outbreak model is initialized with the proportion of the population in each compartment for each city. A set

of differential equations with specified transition state parameters is used to quantify the proportion of the population in each compartment at each point in time, *e.g.*, day. In this work the local model is applied to each city independently. To model the spread of infection between cities we developed a global network model. The nodes of the global network are cities, and the links connect cities based on air traffic data, specifically passenger flow volumes. The spread of disease between cities is modeled as a stochastic process based on the passenger flow volume between city pairs, and the likelihood passengers departing a given city are infected. The final coupled outbreak simulation model captures the evolution of the outbreak due to spread both within and between cities, and yields the number of individuals in each compartment at each point in time in every city in the world. The simulation model is further extended to include a (decision) variable representative of passenger screening at each airport, and thus provides a means to quantify the impact of various screening policies.

The upper level of the model addresses the problem of selecting a feasible control strategy which minimizes the total number of infected individuals at a pre-defined time horizon. In this work a control strategy is defined as a set of airports at which to conduct control, *e.g.*, enhanced passenger screening, which is further specified at a given level. Control level are 2-dimensional variables $x_{i,t}$ indexed by space (airports, denoted by index i) and time (denoted by index t) bounded between 0 and 1, where $x_{i,t} = 1$ corresponds to perfect control of airport i at time t, *i.e.,* all passengers are successfully screened, $x_{i,t} = 0$ corresponds to no screening conducted, and values strictly between 0 and 1 corresponds to partial control, where a fraction of the passengers are screened (or only a fraction of the infectious passengers are successfully identified). Additionally, implementing control at a given airport incurs a cost, which is dependent on both the control level and size of the airport, *i.e.*, number of arriving travelers. In this work we assume a two-part cost function which includes a fixed and variable cost. The fixed cost assumes a one-off cost for training personnel, purchasing and setting up new equipment. The variable cost is assumed to capture the daily cost of conducting passenger screening. The objective is to find the optimal control strategy $\boldsymbol{x}^{\star}$ subject to a budget constraint and accounting for outbreak dynamics.

The proposed problem is challenging due to both the scale of the problem (there are thousands of airports in the world) and the stochastic dynamic nature of disease outbreaks. Thus, the focus of this research is to develop a set of solution methods which are both effective and computationally tractable. In this work a set of proposed heuristics are presented and evaluated. The first set of heuristics are data driven, and exploit known properties of the regional network (*e.g.,* population levels), and air traffic network structure (connectivity indices, etc), while the second set of heuristics utilize outputs from the lower level simulation model to select control strategies. In particular, we propose an algorithmic framework to explore the lower-level solution space efficiently and approximate airports' potential in mitigating the epidemic. This information is then used to construct informed upper-level solutions. All heuristics are evaluated under a range of outbreak scenarios (*i.e.,* source cities, outbreak size, etc.) and disease properties (*i.e.,* level of infectiousness, recovery times, etc). The heuristics performance is quantified using various metrics such as the total number of individuals infected ever infected at a given point in time. The results provide a means to rank outbreak control strategies in terms of performance, cost and reliability. Preliminary results are shown in Figure 1.

In conclusion, this work introduces a novel methodology that captures the stochastic nature of outbreak dynamics within a control policy evaluation framework, necessary for effective regional preparedness planning for large scale outbreaks. The problem framework is

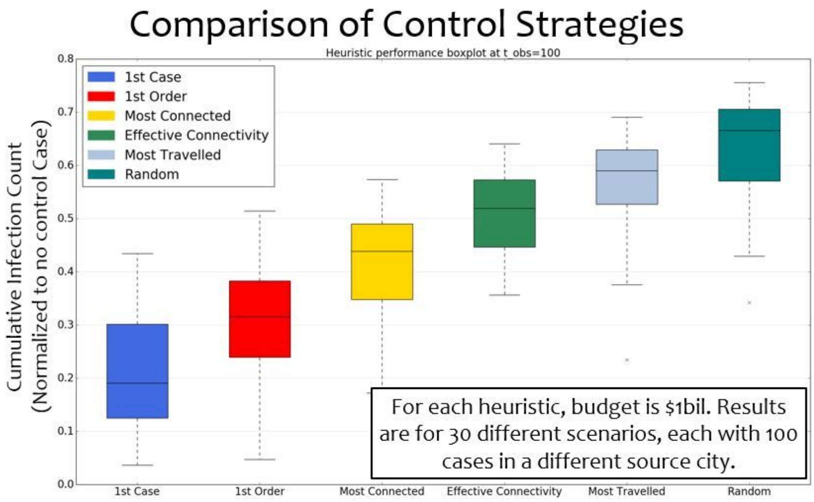


Figure 1. Preliminary Results illustrating the ranking and performance of the proposed heuristics

highly flexible, and applicable to a range of control policies, objectives and outbreak scenarios. Preliminary results provide insight into the robust design and efficiency of a range of control strategies, and can help identify the best use of available budget in the event of a biosecurity episode. It is shown that all proposed allocation strategies out perform the strategy to screen passengers at the largest airports. There is also an observable decrease in the marginal return on investment for the best performing heuristics, thus, identifying optimal control policies is non-trivial. The proposed framework can help in informing policy decisions and resource spending for epidemic planning. Future work will address sensitivity analysis around outbreak parameters, planning horizon and the control cost function, as well as extending the model to a multi-modal mobility network, incorporating local control, and adding a route level control decision.

Keywords: *epidemics, resource allocation, outbreak control*

References

[1] Chen, N., Gardner, L. and Rey, D. *A bi-level optimization model for the development of real-time strategies to minimize epidemic spreading risk in air traffic networks*, Transportation Research Record: Journal of the Transportation Research Board **2569** (2016), 62–69. DOI: 10.3141/2569-07.

[2] Chen, N., Gardner, L. and Rey, D. *A multi-scale network model for evaluating global outbreak control strategies*, Transportation Research Record: Journal of the Transportation Research Board **2626** (2017), 42–50. DOI: 10.3141/2626-06.

[3] Kermack, W. O. and A. G. McKendrick. *A Contribution to the Mathematical Theory of 5 Epidemics*, Proceedings of the Royal Society of London A: Mathematical, Physical and 6 Engineering Sciences **115(772)** (1927), 700–721.

Contributed session

Formulating Bilevel Power Grid Security Applications with Pyomo

William E. Hart[*]
Center for Computer Research, Sandia National Laboratories, wehart@sandia.gov

Bryan Arguello
Energy and Geoscience Center, Sandia National Laboratories, barguel@sandia.gov

John Siirola
Center for Computer Research, Sandia National Laboratories, jdsiiro@sandia.gov

Bismark Singh
Center for Computer Research, Sandia National Laboratories, bsingh@sandia.gov

Jean-Paul Watson
Center for Computer Research, Sandia National Laboratories, jwatson@sandia.gov

Abstract We describe capabilities for modeling bilevel and multilevel programs within the Pyomo modeling software. These capabilities include modeling components that represent subproblems, modeling transformations for re-expressing models with bilevel structure in other forms, and optimize bilevel programs with meta-solvers that apply transformations and then perform optimization on the resulting model. We illustrate the application of Pyomo to model power grid security problems.

Keywords: *algebraic modeling languages, reformulations, power grid security*

1 Introduction

Although multilevel problems arise in many applications, few algebraic modeling languages (AML) have integrated capabilities for expressing these problems. AMLs are high-level programming languages for describing and solving mathematical problems, particularly optimization-related problems. AMLs provide a mechanism for defining variables and generating constraints with a concise mathematical representation, which is essential for large-scale, real-world problems that involve thousands or millions of constraints and variables. GAMS, YALMIP and Pyomo provide explicit support for modeling bilevel programs. A variety of other AMLs support the solution of bilevel programs through the expression of Karush-Kuhn-Tucker conditions and associated reformulations using mixed-complementarity conditions, but these reformulations must be expressed by the user in these AMLs.

The main point of this paper is to describe modeling techniques in Pyomo that can express complex multilevel optimization problems. Pyomo is an open-source software package that supports the definition and solution of optimization applications using the Python language [1]. Python is a powerful programming language that has a clear, readable syntax and intuitive object orientation. Pyomo uses an object-oriented approach for defining models that contain decision variables, objectives, and constraints.

Multilevel models can be easily expressed with Pyomo modeling components for submodels, which can be nested in a general manner. Further, Pyomo's object-oriented design naturally supports the ability to automate the reformulation of multilevel models into other forms. In particular, we describe Pyomo's capabilities for transforming bilevel models for several broad classes of problems, and we illustrate this capability on a power grid security application.

2 Modeling Bilevel Programs

The `pyomo.bilevel` package extends Pyomo by defining a new modeling component: `SubModel`. The `SubModel` component defines a subproblem that represents the lower level decisions in a bilevel program. This component is like Pyomo's `Block` component; any components can be added to a `SubModel` object. In general, a submodel is expected to have an objective, one or more variables and it may define constraints.

The `SubModel` class generalizes the `Block` component by including constructor arguments that denote which variables in the submodel should be considered fixed or variable. When expressions in a submodel refer to variables defined outside of the submodel, the user needs to indicate whether these are fixed values defined by an upper-level problem. Fixed variables are treated as constants within the submodel, but non-fixed variables are defined by the current submodel or by a lower-level problem.

Consider the following example:

$$
\begin{array}{lll}
\min_{x,y,v} & x + y + v & \\
\text{s.t.} & x + v \geq 1.5 & \\
& 1 \leq x \leq 2 & \\
& 1 \leq v \leq 2 & \\
& \max_{y,w} & x + w \\
& & y + w \leq 2.5 \\
& & 1 \leq y \leq 2 \\
& & 1 \leq w \leq 2
\end{array}
\tag{1}
$$

The following Pyomo model defines four variables, `x`, `v`, `sub.y` and `sub.w`:

```
from pyomo.environ import *
from pyomo.bilevel import *

model = ConcreteModel()
model.x = Var(bounds=(1,2))
model.v = Var(bounds=(1,2))
model.sub = SubModel()
model.sub.y = Var(bounds=(1,2))
model.sub.w = Var(bounds=(-1,1))

model.o = Objective(expr=model.x + model.sub.y + model.v)
model.c = Constraint(expr=model.x + model.v >= 1.5)
model.sub.o = Objective(expr=model.x+model.sub.w, sense=maximize)
model.sub.c = Constraint(expr=model.sub.y + model.sub.w <= 2.5)
```

Variables `x` and `v` are declared in the upper-level problem, and `v` only appears in the upper-level problem. Variables `sub.y` and `sub.w` are declared in the submodel. However, note that the `sub.y` variable appears in the upper-level problem, while the `sub.w` variable only appears in the lower-level problem.

3 Bilevel Optimization for Power Grid Security and Resiliency

Expressions in submodels can be linear or nonlinear, convex or nonconvex, continuous or discontinuous, and more. Additionally, submodels can be nested to an arbitrary degree. Thus, the range of bilevel programs that can be expressed with Pyomo is quite broad. However, the real challenge is solving these models.

Pyomo's supports the structured transformation of models, and this capability can be applied to transform bilevel and multi-level problems into standard forms. Pyomo's object-oriented design allows the transformation of sub-models within a formulation. Additionally, blocks of constraints can be activated and deactivated, which facilitates *in place* transformations that do not require the creation of a separate copy of the original model. Thus, bilevel models can be transformed and solved, and the resulting solution can be easily mapped back into the original model.

We describe how these transformations can be applied to reformulate a bilevel formulation that is used identify worst-case security risks for power grid disruptions (e.g. terrorist attacks or vandals). The bilevel formulation is used to identify critical system components by finding the most disruptive attacks given limited attacker resources. The impact of a disruption is derived from a power flow model, which could be weighted (e.g. to balance equities in different regions with different network toplogies). Power flow is modeled with a linear DC-OPF formulation, and transformations can be applied to reformulate this formulation as an integer program.

We describe how these results are used to support an investment model that prioritizes mitigation investments (e.g. to mitigate consequences or reduce likelihood of disruptions). The critical components identified in the bilevel model are prioritized for investments. Finally, we discuss whether trilevel formulations are suitable for the investment planning problem. We describe "nested defense" strategies that can be used to maximize the utility of migitation over a range of possible investment budgets, and discuss the need for multi-year investment planning.

References

[1] Hart, W. E., C. D. Laird, J.-P. Watson, D. L. Woodruff, G. A. Hackebeil, B. L. Nicholson, and J. D. Siirola, *Pyomo: Optimization Modeling in Python*, Springer, 2017.

Influence Maximization with Deactivation in Social Networks

Kübra Tanınmış*
Necati Aras
İ. Kuban Altınel
Department of Industrial Engineering, Boğaziçi University,
{kubra.taninmis,arasn,altinel}@boun.edu.tr

Abstract In this paper we consider an extension of the well-known Influence Maximization Problem (IMP) which deals with finding an initial set of k nodes to initiate a diffusion process in a social network so that the total number of affected nodes at the end of the process is maximized. The extension focuses on a competitive variant of IMP where two decision makers are involved. The first one, the leader, tries to maximize the total influence spread by selecting the most influential nodes and the second one, the follower, tries to minimize it by deactivating some of these nodes. The formulated bilevel model is solved by complete enumeration for small-sized instances and by a matheuristic for large-sized instances.

Keywords: *Influence maximization, Stackelberg game, Stochastic optimization*

1 Introduction

Influence Maximization Problem (IMP) is the problem of finding an initial set of k nodes to start a diffusion process in a social network so that the total number of affected nodes at the end of the process is maximized. It finds many application areas including viral marketing and disease outbreak detection. The most common diffusion models in the IMP literature are Linear Threshold (LT), Independent Cascade (IC), and their extensions. The problem is known to be $\mathcal{NP}$-hard under both models [1], and the LT diffusion model gives rise to a stochastic optimization problem due to the uncertainty in the threshold values of the nodes. The problem considered in this study is a competitive variant of IMP which can be considered as a Stackelberg game. The leader of the game tries to maximize its influence spread by selecting the most influential nodes while the follower tries to minimize the spread by deactivating some of these nodes. Solving the problem of the leader helps to understand which nodes are more prone to be targeted by undesired influence spread initiators.

2 Problem Definition

The problem we address is a two player IMP. Given a directed graph $G(V, A)$, the leader decides a subset $S \subset V$ of nodes to activate in order to initiate campaign. Then, the follower with perfect information on S *deactivates* some nodes $P \subset S$ so that they cannot influence other nodes, which has the consequence of keeping the number of activated nodes at a low level. After the follower's decision, the nodes in $S \setminus P$ influence other nodes in the network according to the well-known Linear Threshold (LT) diffusion model where a node becomes influenced only if the total weight on the incoming arcs from its influenced neighbors exceeds a random threshold value. The goal of the leader is to maximize the expected number of influenced nodes whereas the follower tries to minimize the same performance measure. We refer to the problem as the Influence Maximization Problem with Deactivation (IMPD). Since the node threshold values in the LT model are uncertain, our problem becomes a stochastic optimization problem. However, a deterministic equivalent formulation can be developed by enumerating each possible realization of the node threshold values. We provide the definition of the parameters and decision variables used in the formulation followed by the model.

Parameters:

w_{ij}	:	weight of arc (i, j), 0 if the arc does not exist
θ_{ir}	:	influence threshold of node i in threshold realization $r \in R$
p_r	:	probability of threshold realization $r \in R$
c_i	:	leader's cost of activating node $i \in V$
e_i	:	follower's cost of deactivating node $i \in V$
$\overline{c}$	:	leader's budget to activate nodes
$\overline{e}$	:	follower's budget to deactivate nodes

Decision variables:

X_i	:	1 if node i is activated by the leader; 0 otherwise
Y_i	:	1 if node i is deactivated by the follower; 0 otherwise
U_{ir}	:	1 if node i is influenced in threshold realization r; 0 otherwise

$$z_D^* = \max_{\mathbf{X}} z(\mathbf{X}) \tag{1}$$

$$\text{s.t.} \sum_{i \in V} c_i X_i \leq \overline{c} \tag{2}$$

$$X_i \in \{0, 1\} \qquad i \in V \tag{3}$$

where $z(\mathbf{X})$ is the optimal objective value of the problem

$$z(\mathbf{X}) = \min_{\mathbf{Y}, \mathbf{U}} \sum_{r \in R} \sum_{i \in V} p_r U_{ir} \tag{4}$$

$$\text{s.t.} \sum_{i \in V} e_i Y_i \leq \overline{e} \tag{5}$$

$$Y_i \leq X_i \qquad i \in V \tag{6}$$

$$U_{ir} \geq X_i - Y_i \qquad i \in V, r \in R \tag{7}$$

$$U_{ir} + Y_i \geq \sum_{j \in V} w_{ji} U_{jr} - \theta_{ir} + \epsilon \qquad i \in V, r \in R \tag{8}$$

$$U_{ir}, Y_i \in \{0, 1\} \qquad i \in V, r \in R \tag{9}$$

In the model above, (1)–(3) and (4)–(9) are the leader's upper level problem and the follower's lower level problem, respectively. Their objective functions are the same, i.e., the expected number of influenced nodes over the threshold realization space R, but the sense of optimization is different. Constraint (2) is the activation budget limit of the leader and constraint (5) is the deactivation budget limit of the follower. Constraints (6) ensure that a node cannot be deactivated unless it is activated by the leader. Constraints (7) force the seed nodes to be influenced when they are not deactivated ($X_i = 1, Y_i = 0$). If the total incoming weight to node i from its influenced neighbors exceeds its threshold in realization r and i is not a deactivated node ($Y_i = 0$), then $U_{ir} = 1$ in (8) (i.e., i is influenced in realization r).

3 Solution Method and Results

One method to solve discrete bilevel optimization problems is to either enumerate the decisions of the leader if possible or develop a heuristic method performing a search in the decision space of the leader, and solving the follower's problem to optimality for each solution generated in the upper level problem. Since the thresholds in the LT model follow a continuous uniform distribution, it is not possible to enumerate all realizations or scenarios. The follower's problem can be approximated in this case using the Sample Average Approximation (SAA) method [2] which is based on repeated sampling from the scenario space and solving the problem for each sample.

We propose a Tabu Search (TS) based matheuristic in which the solution space of the leader is searched and for each solution visited the objective value is estimated using the SAA method. Only feasible solutions are considered and same solutions are not visited more than once by means of hash functions due to the costly objective function evaluation. At each iteration, only a promising subset of the neighboring solutions are considered according to a potential influence value computed in a preprocessing step (e.g., if a node has smaller chance to be influenced by the seed nodes, then adding it to the seed improves the current solution more). Three types of move operators (ADD, DROP and SWAP) are used. Each move type selects the best feasible and unprecedented solution in that neighborhood. Table 1 presents the results for small test instances where n and m denote the number of nodes and arcs, respectively. For each size, five problem instances were generated. The second column shows the average optimality gap between the objective value of the solution provided by the TS method and the optimal objective value z^* given by complete enumeration. The last two columns show the solution times in seconds for both methods. The time limit of the TS method is set to three hours. The TS-based matheuristic seems to be promising and can be used for larger instances.

Table 1: Results obtained by the Tabu Search Method and Complete Enumeration

(n, m)	Gap(%)	z^*	TS Time	Enumeration Time
20,40	1.06	8.71	1355	1316
20,80	0.96	11.34	1827	1788
30,90	0.42	17.83	10,800	103,442
30,180	0.00	18.25	10,800	115,485

References

[1] Chen, W., L. V. Lakshmanan and C. Castillo, "Information and Influence Propagation in Social Networks," Synthesis Lectures on Data Management **5**, Morgan & Claypool Publishers, 2013.

[2] Kleywegt, A., A. Shapiro and T. Homem-de-Mello, *The Sample Average Approximation Method for Stochastic Discrete Optimization*, SIAM J. on Optimization **12** (2002), 479–502.

Contributed session

The Bilevel Minimum Spanning Tree

Miguel A. Pozo*
Department of Applied Math II - IMUS, University of Seville,
miguelpozo@us.es

Martine Labbé
Computer Science department, Université Libre de Bruxelles,
mlabbe@ulb.ac.be

Justo Puerto
Department of Statistics and OR - IMUS, University of Seville,
puerto@us.es

Abstract Let be given a graph $G = (V, E)$ whose edge set is partitioned into a set R of red edges and a set B of blue edges, and assume that red edges are weighted and form a spanning tree of G. Then, the Bilevel Minimum Spanning Tree Problem (BMSTP) is that of pricing (i.e., weighting) the blue edges in such a way that the total weight of the blue edges selected in a minimum spanning tree of the resulting graph is maximized.

In spite of its interest, no efficient formulations exist for the BMSTP and no mathematical algorithms have been developed for solving the problem. In this paper we will present different MILP mathematical formulations for the BMSTP based on the properties of the MSTP and the bilevel optimization. We establish a theoretical and empirical comparison between these new formulations and we also provide reinforcements that together with a proper formulation are able to solve medium size random instances on general graphs.

Keywords: *Minimum Spanning Tree, Bilevel optimization, Stackelberg game*

A brief discussion regarding the bi-level semi-feasible solutions

José-Fernando Camacho-Vallejo *
Facultad de Ciencias Físico-Matemáticas, Universidad Autónoma de Nuevo León,
`jose.camachovl@uanl.edu.mx`

Abstract It is well-known that for obtaining feasible solutions of a bi-level programming problem, the optimal solution of the lower level is needed. But achieving the latter is complicated when the lower level is a very complex problem or it is classified as NP-hard. In some of these cases obtaining bi-level feasible solutions may be impossible. Hence, what options do we have for dealing with such difficult problems? In this talk, some related literature is discussed pointing out the major conclusions and drawbacks found. Also, the appropriateness of the semi-feasible solutions in some cases is discussed.

Keywords: *Bi-Level Programming, semi-feasible solutions, heuristic solutions*

1 Background

In the first book devoted to bi-level programming, [1], it is stated that for a fixed leader's solution, the follower acts rationally and optimizes its own criterion. Hence, the resulting pair of leader-follower solutions corresponds to a bi-level feasible one. Most of the papers devoted to study bi-level problems handle this issue in a proper manner. However, there are few papers in which the lower level is a hard-to-solve problem. In those cases, the authors have dealt with it in a particular manner for each problem.

In a personal discussion I had with Professor Bard few years ago about this issue, his answers were based on the argue that the follower reacts rationally to leader's decision. Hence, if the follower has a very complex problem and obtaining the optimal solution is a difficult task, it is rational to consider that a *good enough* solution is allowed. Since then, I have put special attention about this topic. Later, in [2] the bi-level semi-feasible solutions are properly defined, and in [3] the bi-level attainable solutions are defined. Both definitions are very similar but they differentiate from each other in a particular characteristic.

2 Objectives of this discussion

This talk has two main objectives. The first one is to show some existing papers that consider bi-level semi-feasible solutions and the manner they handled with the lower level problem. Furthermore, the impact of not having the optimal response of the follower for a leader's decision is illustrated with numerical examples. Therefore, it is emphasized the careful that is needed when dealing with this issue in bi-level problems.

The other objective is to start a discussion with the attendees regarding the options for solving this kind of problems without loosing mathematical rigour, nor relaxing or simplifying the studied bi-level problem.

References

[1] Bard, J.F., 1998. Practical Bilevel Optimization: Algorithms and Applications, Kluwer Academic Publishers, Dordrecht, The Netherlands

[2] Alekseeva, E., Kochetov, Y., 2013. Metaheuristics and Exact Methods for the Discrete (r/p)-Centroid Problem, in Talbi, E.-G. (Ed) Metaheuristics for Bi-Level Optimization, Springer, Berlin

[3] Casas-Ramírez, M.S., Camacho-Vallejo, J.F., Martínez-Salazar, I.A., 2017. Approximating solutions to a bilevel capacitated facility location problem with customer's patronization toward a list of preferences, Applied Mathematics and Computation.

S

Generic properties of single-leader-multi-follower games

Gemayqzel Bouza-Allende *

Department of Applied Mathematics, University of La Habana , gema@matcom.uh.cu

Abstract Nowadays many applications involve decisions that depend on the optimal decisions of other agents. This kind of model has a bi-level structure with many lower level problems and it is known as single-leader-multi-follower game. An application in eco-park can be found in [2].

In this contribution we substitute the lower level optimization problems by the stationarity conditions and consider the resulting mathematical program with complementarity constraints (MPCC) formulation. We analyze which properties will remain stable under perturbations of the involved functions, as it was done in [1] for bilevel problems. Then, the relation of the stationary points with these properties and the solutions of the original problem are presented. Particular cases are analyzed.

Keywords: *single-leader-multi-follower game, stability, stationary points*

References

[1] Bouza Allende, G. and G. Still, Solving bilevel programs with the KKT-approach Math. Program., Ser. A **138** (2013) 309-332.

[2] Ramos, M. A., M. Boix, D. Aussel, L. Montastru, S. Domenech. Water integration in eco-industrial parks using amulti-leader-follower approach, Computers and Chemical Engineering, **87** (2016) 190â207

On a Multi-leader-follower model for electricity market with elestic demand

Didier Aussel*
Laboratory PROMES UPR CNRS 8521, University of Perpignan, France,
aussel@univ-perp.fr

Elisabetta Allevi
of Economics and Management, Università degli Studi di Brescia, Italy,
elisabetta.allevi@unibs.it

Rossana Riccardi
of Economics and Management, Università degli Studi di Brescia, Italy,
rossana.riccardi@unibs.it

Abstract The present talk is based on Allevi-Aussel-Riccardi[**?**] and can be considered as an extension to the case of an elastic demand of the analytical study initiated in [**?**, **?**]. We consider a model of pay-as-clear electricity market modeled as a multi-leader-common-follower game where the lower level represent the regulator of the market (ISO). One of our aim in this work is to analyse the best response problem of each producer when the demand is actually composed of a fixed part coupled with an elastic quantity. We also prove that an equilibrium of the market exists under mild assumptions on the structure of the bid and on the elasticity. Some numerical experiments on a simplified market model are also provided.

Keywords: *Multi-leader-follower game, electricity market, Elastic demand*

References

[1] E. Allevi, D. Aussel & R. Riccardi, *On a equilibrium problem with complementarity constraints formulation of pay-as-clear electricity market with demand elasticity*, J. Global Optim. (2018), 15 pp.

[2] Aussel D., Bendotti P.and Pištěk M., *Nash equilibrium in a pay-as-bid electricity market: Part 1 - existence and characterization*, Optimization, **66(6)**, (2017).

[3] Aussel D., Bendotti P.and Pištěk M., *Nash equilibrium in a pay-as-bid electricity market: Part 2 - best response of a producer*, Optimization, **66(6)**, (2017)..

Two-Level Value Function Approach to Nonsmooth Optimistic Bilevel Programs

Stephan Dempe*
Department of Numerical Mathematics and Optimization, TU Bergakademie Freiberg, Germany , `dempe@tu-freiberg.de`

Boris S. Mordukhovich
Department of Mathematics, Wayne State University, Detroit, U.S.A. , `boris@math.wayne.edu`

Alain B. Zemkoho
School of Mathematics, University of Southampton, UK, `a.b.zemkoho@soton.ac.uk`

Abstract The optimistic (pessimistic) bilevel optimization problem can be modelled as minimizing the optimistic (resp. pessimistic) two-level optimal value function:

$$\varphi_o(x) := \min_y \{F(x,y) : y \in \Psi(x)\} \to \min_{x \in K},$$

resp.

$$\varphi_p(x) := \max_y \{F(x,y) : y \in \Psi(x)\} \to \min_{x \in K},$$

where

$$\Psi(x) = \{y : g(x,y) \leq 0,\ f(x,y) \leq \varphi(x)\}$$

denotes the set of optimal solutions and $\varphi(x) = \min_y\{f(x,y) : g(x,y) \leq 0\}$ is the optimal value function of the lower level problem. Topic of the presentation is the formulation of necessary optimality conditions for the optimistic problem using tools from variational analysis [2] in the case when the bilevel optimization problem is formulated with nonsmooth data. This generalizes the necessary optimality conditions for continuously differentiable optimistic bilevel optimization problems presented in [1].

Keywords: *optimistic and pessimistic bilevel programming, two-level value functions, variational analysis, generalized differentiation, optimality conditions.*

References

[1] S. Dempe, B.S. Mordukhovich and A.B. Zemkoho, *Sensitivity analysis for two-level value functions with applications to bilevel programming,* SIAM J. Optim. 22 (2012), 1309 - 1343.

[2] B.S. Mordukhovich, *Basics of Variational Analysis and Applications,* Springer, New York, 2018.

The negative $(r, p) - centroid$ problem considering loyalty of the customers

Juan Carlos García Vélez*
Facultad de Ciencias Físico-Matemáticas, Universidad Autónoma de Nuevo León,
juan.garciavlz@uanl.edu.mx
José-Fernando Camacho-Vallejo
Facultad de Ciencias Físico-Matemáticas, Universidad Autónoma de Nuevo León,
jose.camachovl@uanl.edu.mx
Juan A. Díaz
Departamento de Física, Actuaria y Matemáticas, Universidad de las Américas Puebla,
juana.diaz@udlap.mx
Diego Ruiz-Hernández
Departamento de Métodos Cuantitativos, Colegio Universitario de Estudios Financieros,
d.ruiz@cunef.es

Abstract The competitive plant location problem is of current interest to the scientific community. Several recently published articles about this kind of problems can be found in the literature. However, most of the papers focus on two competing companies that aim to locate facilities (r and p, respectively) and to capture the largest proportion of the market. This sequential problem is known in the literature as the $(r|p) - centroid$. But there are different situations in various social, political, economic and financial fields that may require that some facilities of the existing ones are closed. The literature on these types of problems is limited. Therefore, we define the problem of the negative $(r|p) - centroid$ where both companies aim to close facilities that are currently located. Another novelty of this work is that a loyalty of the customers towards the facilities is considered to make the allocation. That is, customers are not assigned based on the shortest distance criterion. In addition, to solve the problem an adaptation of a classical branch and bound algorithm proposed by [1] is proposed. The computational experimentation indicates that the adapted algorithm is able to solve problems efficiently and at low computational cost.

Keywords: *Competitive Facility Location, Bilevel programming, Delocation, (r/p)-centroid problem.*

1 Introduction

Facility location problems (FLP) are a branch of operations research that arise when real-life applications in the public and private sector attempt to determine the optimal location of facilities, such as warehouses, plants, distribution centers, shopping centers, hospitals, post offices, among others. In these types of problems, you may have different objectives such as maximizing the benefits obtained from customers and minimizing costs when locating facilities and serving customers. Many factors influence the decision to locate a new facility in a market, but one of the most important factors is related to the existing facilities that belong to competitors that offer the same or similar services or products. When there is no competitor in the market, the new facility will be the only supplier of the merchandise or the service in the market, however, if there are already facilities in the market, the new facility will have to compete for the customers aiming to maximize its profit or market share.

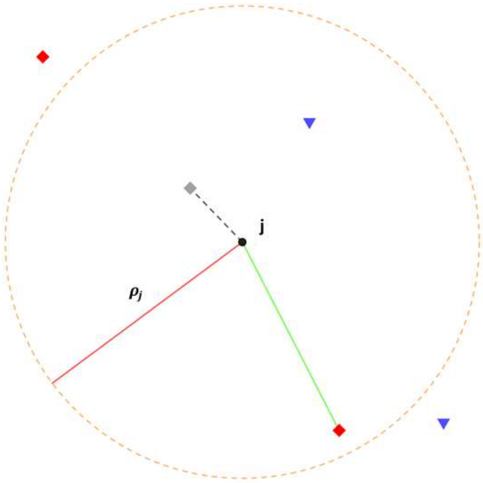
The Competitive Facility Location problem (CFL) takes place when two or more companies try to enter the market, competing among them to capture the largest proportion of customers, offering a product or service. This type of problems have been considered from a bilevel (leader-follower) approach, since they have a natural bilevel programming model. For example, [2] examined sequential competitive location problems. They divided the problems into two categories: the $(r|X_p) - medianoid$ problem, in which the leader has p facilities, and the follower optimally locates r facilities of a set X_p of potential sites; and the $(r|p) - centroid$ problem, in which the leader locates optimally p facilities knowing that the follower will react locating r facilities. In the latter problem, there are two firms that try to locate facilities in a hierarchical way, aiming to maximize their market share (demand); in other words, capture most of the customers. There is a set of N potential sites, from which the leader locates p facilities and then from the $N - p$ remaining potential sites the follower selects his/her r facilities.

2 Problem Statement

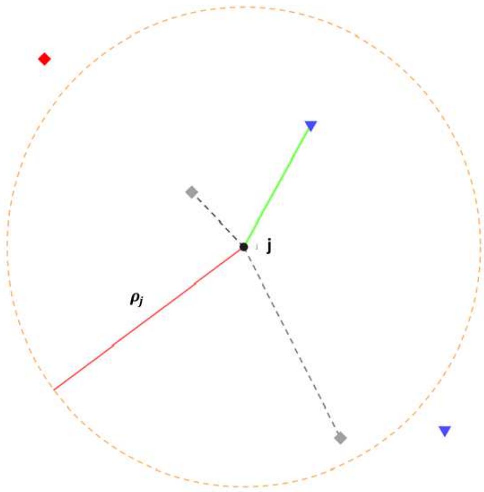
Nowadays some markets are prone to suffer an economic contraction caused by some particular issues. So, when there is an economic downturn the companies are faced with the problem of not being able to keep all of their facilities operating and have the need to close some of them. Literature of this type of problems on offshoring (delocation) of competitive facilities is limited. After an intensive search, only [3] was found.

The problem herein proposed seeks to close already located facilities taking into account the possible actions of rival companies and the reaction of the customers. We named this problem as the negative $(r|p) - centroid$ problem, and it is denoted as $\overline{(r|p)} - centroid$. An additional variant is that we consider the assignment of customers to the facilities by taking into account the loyalty of the customer towards the same company to which it is currently assigned. That is, it is not necessarily assigned to the nearest located facility, but remains with the same company as long as there is another facility within a predetermined threshold value (see figure 1).

This problem is modelled as a as a bilevel program, where first the leader (upper level) closes p facilities and then, the follower (lower level) closes r facilities, which already are operating in the market. Also, the corresponding valid constraints that allocates the customers based on their loyalty are added.



(a) Example of a loyal customer, stays with the same company



(b) Customer captured by the competition

Figure 1: Loyalty of a customer $j - \bullet$ customer loyalty to the company that he is loyal within a radius ρ_j. In gray, a closed installation is denoted.
◆- Leader's facility ▼- Follower's facility

3 Proposed algorithm

A first scheme to solve the $\overline{(r|p)}$-centroid is based on the algorithm proposed in [1]. This algorithm solves binary bilevel programming problems using a branch and bound scheme. The idea is to list the nodes of a binary tree associated with the problem of the leader but taking into account the optimal reaction of the follower. That is, the algorithm tries to find the best decision of the leader among the feasible bilevel solutions in the inducible region. Particular properties of the problem are considered and used to improve the algorithms performance.

Computational experimentation have been conducted for testing the algorithm over a set of randomly created instances. The proposed algorithm has shown to be a good option to solve the $\overline{(r|p)}$-centroid problem. The optimal solutions are found using a low computational cost. To measure the quality of the solution, a totally enumerative was used.

References

[1] Bard, Jonathan F. and Moore, James T. (1992). An algorithm for the discrete bilevel programming problem. *Naval Research Logistics (NRL) (1992), 419–435.*

[2] Kress. D., and Pesch, E. (2012). Sequential competitive location on networks. *European Journal of Operational Research, 217(3), 483-499.*

[3] Ruiz, D., Elizalde, J., Delgado D. (2016). Cournot-Stackelberg games in competitive delocation. *Annals of Operations Research(2016), DOI 10.1007/s10479-016-2288-z.*

Contributed session

A bilevel formulation of a choice-based location model involving competition and queueing

Teodora Dan*
Department of Computer Science and Operations Research, University of Montreal,
teodora.dan@umontreal.ca

Patrice Marcotte
Department of Computer Science and Operations Research, University of Montreal,
marcotte@iro.umontreal.ca

Abstract

We consider the problem faced by a service firm that makes decisions with respect to location and service levels of its facilities, when users patronize the facility minimizing the sum of travel time, queueing delay, and a random term. This situation can be modeled as a bilevel program that involves discrete and continuous variables, as well as linear and nonlinear functions. We design for its solution an approximation algorithm that provides asymptotically optimal solutions, as well as heuristics that exploit the very structure of the problem.

Keywords: *location, bilevel programming, equilibrium, queueing, nonconvex*

1 Introduction

While the literature concerning discrete facility location is vast, few studies have focused on user choice, where the latter frequently involves congestion, either along the paths leading to a facility, or at the facility itself. Our aim is to provide a model that captures the key features of congestion and competition within a user choice environment, yielding a bilevel program where the leader firm's objective function integrates the stochastic equilibrium resulting from the choice of locations and the associated service levels.

Beyond the analysis of the model's theoretical properties, we designed two algorithms, one based on approximations, and a heuristic. Our work is closely related to that of [1], who analyze a location model where queueing is explicitly taken into account, while users are assigned to facilities according to a logit discrete choice model, yielding a mathematical program involving user-equilibrium constraints. Our model is well suited to a variety of applications, such as location of shops, restaurants, walk-in clinics, etc., where user flows are not in direct control of the optimizer, but are dictated by utility maximization principles.

2 Model

Let us consider the problem faced by a firm (a service center, for instance) that makes location and service level decisions in a bipartite network $V = I \times J$, where a vertex v may correspond to either a location (J) or a demand node (I), the latter endowed with demand d_v. We denote by $J_1 \subseteq J$ the set of candidate locations for firm A, and by J_c the set of locations of its competitors. The aim of the emerging firm is to maximize the number of customers to attract, constrained by a budget B that can be spent on building facilities or improving service rate. Facilities are modeled as $M/M/1/K$ queues with the waiting time $w(\lambda, \mu)$ and probability of balking $p_K(\lambda, \mu)$.

A salient feature of the model is that user behavior is explicitly taken into account. Users originating from node i patronize the facility j that maximizes their individual utility $\tilde{u}_{ij}$, estimated as the sum of travel time to the facility (t_{ij}), queueing at the facility (w_j), plus the probability of balking (p_{Kj}):

$$\tilde{u}_{ij} = -\underbrace{(t_{ij} + \alpha w_j + \beta p_{Kj})}_{u_{ij}} + \varepsilon_{ij}, \tag{1}$$

where ε_{ij} are independent Gumbel variates with comon scale parameter θ and variance $\pi^2/(6 \cdot \theta^2)$. In this multinomial logit framework (see [2]), the demand generated at node i that patronize an open facility j is given by the expression

$$x_{ij} = d_i \frac{e^{-\theta\,(t_{ij} + \alpha w_j + \beta p_{Kj})}}{\sum_{l \in J^*} e^{-\theta\,(t_{il} + \alpha w_l + \beta p_{Kl})}}, \tag{2}$$

where J^* represents the set of open facilities. For small values of θ, users are spread more or less evenly between facilities while, when θ is large, the assignment approaches that of a Wardrop equilibrium (see [3]). According to our assumptions, the problem can be formulated as the equilibrium-constrained nonlinear mixed integer program involving a leader and a follower (users):

$$
\begin{array}{lll}
\text{(P) LEADER:} & \max\limits_{\substack{y,\mu,\lambda,\bar{\lambda},\\ x,w,p,\rho}} & \sum\limits_{j \in J_1} \bar{\lambda}_j \\
& & \sum\limits_{j \in J_1} c_f y_j + \sum\limits_{j \in J_1} c_\mu \mu_j \leq B, \\
& & \mu_j \leq M y_j, \qquad \forall j \in J_1 \\
& & \bar{\lambda}_j = \lambda_j (1 - p_{Kj}), \qquad \forall j \in J \\
& & y_j \in \{0,1\}, \mu_j \geq 0, \qquad \forall j \in J_1 \\
\text{USERS:} & & x_{ij} = d_i \dfrac{y_j \cdot e^{-\theta\,(t_{ij} + \alpha w_j + \beta p_{Kj})}}{\sum_{l \in J^*} e^{-\theta\,(t_{il} + \alpha w_l + \beta p_{Kl})}}, \qquad \forall i \in I;\ \forall j \in J \\
& & \lambda_j = \sum\limits_{i \in I} x_{ij}, \qquad \forall j \in J \\
& & \rho_j \mu_j = \lambda_j, \qquad \forall j \in J \\
& & p_{Kj} = \rho_j^K \dfrac{1 - \rho_j}{1 - \rho_j^{K+1}}, \qquad \forall j \in J \\
& & w_j(\lambda_j, \mu_j) = \dfrac{1}{\mu_j} \left(K + \dfrac{K}{\rho_j^K - 1} - \dfrac{1}{\rho_j - 1} \right), \qquad \forall j \in J.
\end{array}
$$

3 Algorithms

We have designed two algorithms for addressing the bilevel location problem.

The first approach is based on a piecewise-linear approximation of non-linear (and non-convex) functions, in order to obtain a linear bilevel problem that can be further reduced to a MILP, through the following five operations:

1. Approximate the lower-level objective function by a piecewise linear approximation.
2. Write the KKT optimality conditions of the lower-level linear program to obtain a single-level mathematical program involving complementarity constraints (MPEC).
3. Formulate the MPEC as an MILP, through the introduction of binary variables and 'big-M' constants.
4. Solve the resulting MILP for optimum values of μ and y.
5. Solve the original nonlinear lower-level problem to recover the true values of the assignment vector x associated with μ and y.

Whenever the approximation is fine-grained, we expect its solution to be close to optimal. We prove that in the absence of balking, this algorithm is asymptotically exact. However, when balking is present, it becomes a matheuristic, and no bound on the error is guaranteed.

The second algorithm is a parameterized heuristic based on a surrogate problem. We replace the original model by a single-level model involving a surrogate objective, whose optimal solution automatically satisfies the fixed point constraint. The rationale behind this strategy is that both the leader and the users have a shared interest in minimizing delays. We therefore expect that, if the lower level is given full control, the resulting design should favor access to the leader's facilities, and therefore yield a high throughput. Our experiments show that this method yields good quality feasible solutions within a reasonable amount of time.

References

[1] Marianov, V., and M. Ríos and M. J. Icaza *Facility location for market capture when users rank facilities by shorter travel and waiting times*, European Journal of Operational Research **191** (2008), 32–44.

[2] McFadden, D. , *Conditional Logit Analysis of Qualitative Choice Behavior*, Frontiers in econometrics (1974),105–142.

[3] Fisk, C. , *Some developments in equilibrium traffic assignment methodology*, Transportation Research B **14**(1980),243–256.

Local search approach for the $(r \mid p)$-centroid problem under l_1 metric

Ivan Davydov*
Department of Mechanics and Mathematics, Novosibirsk State University, Russia
Sobolev Institute of Mathematics, Novosibirsk, Russia, vann.davydov@gmail.com

Yury Kochetov
Department of Mechanics and Mathematics, Novosibirsk State University, Russia
Sobolev Institute of Mathematics, Novosibirsk, Russia, jkochet@math.nsc.ru

Petr Gusev
Department of Mechanics and Mathematics, Novosibirsk State University, Russia, pgusev@nsu.ru

Abstract In the $(r \mid p)$-centroid problem, two players, called the Leader and the Follower, open facilities to service customers. We assume that customers are identified with their location on the plane, and facilities can be opened anywhere in the plane. The Leader opens p facilities. Later on, the Follower opens r facilities. Each customer patronizes the closest facility. The distances are calculated according to l_1-metric. The goal is to find the location of the Leader's facilities maximizing her market share. We provide the results on the computational complexity of this problem and develop a local search heuristic, based on the VNS framework. Computational experiments on the randomly generated test instances show that the proposed approach performs well.

1 Introduction

This paper addresses a Stackelberg facility location game on a two–dimensional plane. We assume that the customers demands are concentrated at a finite number of points in the plane. In the first stage of the game, a player, called the Leader, opens p facilities. At the second stage, another player, called the Follower, opens r facilities. At the third stage, each customer chooses the closest opened facility as a supplier. We consider the case when the distances are defined according to l_1-metric. In case of ties, the Leader's facility is preferred. Each player tries to maximize its own market share. The goal of the game is to find p points for the Leader facilities to maximize her market share. We assume that the Leader knows how many facilities the Follower will locate.

The $(r \mid p)$-centroid problem was first studied by Hakimi for location on a network. A comprehensive review of complexity results and properties of the problems can be found in [1].

In this paper we present a local search heuristic using an exact approach for the Follower problem. We consider the $(r \mid X_{p-1} + 1)$-centroid problem where the Leader moves exactly one facility. We use this problem in order to find the best neighboring solution in the swap neighborhood. To reduce the computational efforts we use the concept of randomized neighborhoods and apply a local descent algorithm to evaluate the goal function during the neighborhood exploration.

2 Mathematical model

Let us consider a two–dimensional plane in which n customers are located. We assume that each customer j has a positive demand w_j. Let X be the set of p points where the Leader opens her own facilities and let Y be the set of r points where the Follower opens his own facilities. The l_1 distances from customer j to the closest facility of the Leader and the closest facility of the Follower are denoted as $d(j, X)$ and $d(j, Y)$, respectively. Customer j prefers Y over X if $d(j, Y) < d(j, X)$ and prefers X over Y otherwise. By

$$U(Y \prec X) := \{j \mid d(j, Y) < d(j, X)\}$$

we denote the set of customers preferring Y over X. The total demand captured by the Follower by locating his facilities at Y while the Leader locates her facilities at X is given by

$$W(Y \prec X) := \sum(w_j \mid j \in U(Y \prec X)).$$

For given X, the Follower tries to maximize his own market share. The maximal value $W^*(X)$ is defined to be

$$W^*(X) := \max_{Y,|Y|=r} W(Y \prec X).$$

This maximization problem will be called the *Follower problem.* The whole market is divided between competitors, so the Leader tries to minimize the market share of the Follower. This minimal value $W^*(X^*)$ is defined to be

$$W^*(X^*) := \min_{X,|X|=p} W^*(X).$$

For the best solution X^* of the Leader, her market share is $\sum_{j=1}^{n} w_j - W^*(X^*)$. In the $(r \mid p)$*–centroid problem,* the goal is to find X^* and $W^*(X^*)$.

We claim that the $(r \mid p)$-centroid problem under l_1 metric is Σ_2^p-hard, while the Follower's problem is $NP-hard$.

3 Local search algorithm

At the first step we address the Follower problem (lower level problem). In this case the location of Leader's facilities in known and given as an input. Together with the locations of customers this information defines the set of attractive regions, one for each customer. Each region represents a set of points on the plane, which are closer to the customer than the closest facility of the Leader. If the Follower opens his facility inside of this region, he captures the corresponding customer. Intersecting regions allows to capture more than one customer simultaneously. The l_1 metric allows us to formulate the problem of enumeration of all the valuable intersections as a problem of finding all maximal cliques in a graph on n vertices. We design a polynomial time procedure, based on the all maximal cliques generation approach [3], in order to provide a list of intersection regions. Using this list, we provide an ILP formulation for the Follower problem. In order to tackle the upper level problem we propose a VNS-based approach. To explore the neighborhood we consider the $(r|X_{p-1}+1) - centroid$ problem where the Leader has a set of $p-1$ facilities and wants to open another facility in the best position [2]. We claim that there is a relatively small number (bounded by polynomial of n) of candidate points in the plane which should be checked in order to find the best one. The proposed approach was implemented in $C\#$ environment and tested on benchmark instances with up to 100 customers for values of $p = r = 5, 10, 15$. Computational results show the effectiveness of the method.

Keywords: *Stackelberg game, Manhattan distance, Matheuristic, Bi-level programming*

References

[1] Kress, D. and E. Pesch, *Sequential competitive location on networks*, European J. Oper. Res. **217** (2012), 483–499.

[2] Kochetov, Y., Davydov, I. and E. Carrizosa , *A local search heuristic for the* $(r|p)$*-centroid problem in the plane*, Computers & Operations Research **52** (2014), 334–340.

[3] Tomitaa, E., Tanakaa, A. and H. Takahashia,*The worst-case time complexity for generating all maximal cliques and computational experiments*, Theoretical Computer Science **363** (2006), 28–42.

Bilevel programming models for multi-product location problems

Sebastián Dávila *
Departament of Industrial Engineering , Universidad de Chile, Chile,
`sdavila@ing.uchile.cl`

Martine Labbé
Departément d'Informatique, Université Libre de Bruxelles, Belgium, `mlabbe@ulb.ac.be`

Frédéric Semet
CRIStAL Centre de Recherche en Informatique Signal et Automatique de Lille, France,
`frederic.semet@centralelille.fr`

Vladimir Marianov
Department of Electrical Engineering, Pontificia Universidad Católica de Chile, Santiago, Chile, `marianov@ing.puc.cl`

Fernando Ordoñez
Departament of Industrial Engineering , Universidad de Chile, Chile,
`fordon@dii.uchile.cl`

Abstract A retail firm has several malls with a known location. A particular product, e.g., food processor, comes in p types, which differ by shapes, brands and features. The set of all p products is P. Each mall j has a limited capacity c_j of products in P to be sold at that location, so the firm has to choose what products to sold at what mall. Furthermore, the firm can apply discrete levels of discount on the products, e.g., 5% and 10% over the price π_k of product k. The objective of the firm is to find what products to sell at which mall, with what level of discount, so that its profit is maximized. Consumers are located in points of the region. Each consumer or group of consumers i has a different set $P_i \subseteq P$ of acceptable products, and will purchase one of these, or none if it is not convenient for her. Consumers maximize their utility, defined as $u_{ijkl} = r_{ik} - \alpha_{jkl} \cdot \pi_{jk} - 2d_{ij}$ where r_{ik} is the maximum expenditure that customer i is willing to make to acquire product k; α_{jkl} is (100 - discount level l in percent) of the product k in mall j; π_{jk} is the price of the product k in the mall j and d_{ij} is the distance between consumer i's origin and mall j. Whenever this utility is negative for product k at all malls, the consumer does not purchase the product. The agents (firm and consumers) play a Stackelberg game, in which the firm is the leader and the customers the follower. Once the firm decides the products to sell at each mall and the possible discounts, consumers purchase (or not) one of their acceptable products wherever their utility is maximized. We model the problem using first a bilevel formulation, and we further replace the follower problem by the primal constraints and optimality restrictions, to obtain a compact formulation. We also present a strong and a weak formulation. Computational experience is offered, using known

instances from the literature [1].

Keywords: *Stackelberg game, multi-product location, bilevel programming*

References

[1] Fernandes, C. G., Ferreira, C. E., Franco, A. J., & Schouery, R. C. (2016). *The envy-free pricing problem, unit-demand markets and connections with the network pricing problem. Discrete Optimization*, **22**, 141-161.

MOGASI: nested bi-level application to NPP

Stefano Costanzo*
Esteco SpA, Area Science Park, Trieste, Italy, costanzo@esteco.com

Desirée Rigonat
DIA, University of Trieste, Italy, desiree.rigonat@phd.units.it

Lorenzo Castelli
DIA, University of Trieste, Italy, castelli@units.it

Abstract Even though bi-level programming problems are difficult to solve [1] they have become relevant in the industrial sector. In fact, many real-world problems in areas such as management, economic planning or engineering involve a hierarchical relationship between two decision levels. In this work we present an application study on the Network Pricing Problem (NPP) [2]. The NPP reflects the decision-making process of a company that (i) has to determine the price of its services to maximize its revenues and (ii) has to consider the customers' reactions to these prices as they may refuse to buy a product or service if the price is too high. This class of problems was first studied in the 1990s and it is NP-hard, although there are polynomial algorithms applicable to particular cases [3] [4].

In this work, we propose to use the Multi-Objective Genetic algorithm Structured Input (MOGASI) algorithm to solve the NPP [5]. This algorithm combines modules and operators of standard Genetic Algorithms (GAs) with specialized routines aimed at achieving enhanced performance on specific types of problems [6]. MOGASI classifies variables and constraints to apply specialized data handling strategies to sub-problems with different data structures. An evolutionary bi-level nested approach is applied to the road network NPP, where typically an authority owns a subset of toll paths and imposes tolls on them in an attempt to maximize revenues. The users traveling on that network seek to minimize costs. This is possible owing to the low computational effort required for finding the best solution for the users. MOGASI is applied at the authority level and an exact solver at the user level. The user problem is rather simple once the authority's decision variables are fixed because it is reduced to a minimum cost assignment problem.

We analyzed the possible differences between the solution with the proposed evolutionary bi-level nested approach and the exact solver solution of a single-level problem reformulation as in [7]. The main focus is on the behavior of the two approaches in relation to the increasing problem size. For this purpose we used a set of random instances introduced in [8] to test product pricing problems, and later used by [9] to compare the NPP formulation and the product pricing formulation for solving product pricing problems. Two categories of instances are generated, i.e., complete and partial networks, to test the proposed optimization framework. Complete network means that toll free path costs are randomly generated between 20 and 30 (and so are always larger than fixed costs of toll paths, which range between 1 and 10 to be summed up with the toll path cost), such that all commodities could use all toll paths, whilst for a partial network the set of toll free path costs is randomly generated between 10 and 30, meaning that some toll paths will never be interesting for some commodities. For both categories a set of five different problem dimensions is taken into consideration, i.e., instances with 20, 56, 90, 132 and 180 commodities and toll paths. The problem is rather difficult and the existing exact approaches are not able to solve large instances within a reasonable amount of time, that is one hour. The idea was to identify a promising area, or at least a

niche, in which a nested meta-heuristic such as MOGASI can compete against or even surpass the performance of traditional exact solvers.

A specific promising area was found for the proposed nested approach with respect to the exact solver. The results presented in this work show that MOGASI is outperformed by the exact solver in small-size instances but the situation is the opposite as the instance size increases. In spite of the fact that both dimensions, i.e. number of toll paths and number of commodities, contribute to the problem complexity. The number of commodities seems to have a larger impact on the exact solver performance. An analysis of the behavior of the MOGASI algorithm shows difficulties to reach convergence in relation to instances with a low number of commodities. A hypothesis is that it is more difficult for the GA to solve problems with a low number of commodities because their distribution implies smaller shifts in the authority objective function values, which MOGASI is tasked with solving. The general performance trends in the 90-, 132- and 180-toll instances for partial network are very positive for the GA for the cases with the highest number of commodities. In particular, for the largest instance sizes the application of MOGASI yielded results of the same quality as those obtained with the exact solver in at least one order of magnitude of time less. For example, in 180-toll/180-commodity partial instance set the MOGASI runs on average for approximately 750s against 3,600s of the exact solver. However, due to the stochastic nature of the GA, this performance cannot always be guaranteed, even though the failure probability is low. Considering that the average execution time of the case with the longest run is approximately a fifth of the time required by the exact solver, we believe that repeated runs are acceptable in the search for the optimum.

Future work will focus on the dynamics of large scale instances to delimit more precisely the promising application area for MOGASI. We also intend to develop new specialized MOGASI modules for integer problems since the current implementation does not have any specialized strategy for handling them, despite the results achieved in this application.

Keywords: *Pricing Problem, Bi-level optimization, Genetic Algorithms*

References

[1] Dempe, S. , *Foundations of bilevel programming*, Nonconvex optimization and its applications, Vol. 61, Kluwer Academic Publishers, 2002.

[2] Labbé M., Marcotte P. and Savard G., *A Bilevel Model of Taxation and Its Application to Optimal Highway Pricing*, Management Science 44(12): 1608-1622, 1998.

[3] Labbé M. and Violin A. , *Bilevel Programming and Price Setting Problems*, 4OR Quarterly Journal of the Belgian, French and Italian Operations Research Societies, 2013.

[4] Castelli L., Labbé M. and Violin A. , *Network pricing problem with unit toll*, Networks 69, pp 83-93, 2017.

[5] Costanzo S., *An Evolutionary Multi-Objective Optimization Framework for Bi-level Problems*, PhD thesis, University of Trieste, Italy, 2016.

[6] Costanzo S., Castelli L. and Turco A. , *MOGASI: A multi-objective genetic algorithm for efficiently handling constraints and diversified decision variables*, Engineering Optimization, pp 99-104, CRC Press-Balkema, 2014.

[7] Violin A., *Mathematical programming approaches to pricing problems*, PhD thesis, Université Libre de Bruxelles and University of Trieste, 2014.

[8] Shioda R., Tunçel L. and Myklebust T., *Maximum utility product pricing models and algorithms based on reservation price*, Computational Optimization and Applications, 48(2): 157-198, 2011.

[9] Heilporn G., Labbé M., Marcotte P. and Savard G., *Valid inequalities and branch-and-cut for the clique pricing problem*, Discrete Optimization, 8(3): 393-410, 2011.

ε-Constraint method for bi-objective competitive facility location problem with uncertain demand scenario

Andrey Melnikov*
Sobolev Institute of Mathematics; Department of Mechanics and Mathematics, Novosibirsk State University, melnikov@math.nsc.ru

Vladimir Beresnev
Sobolev Institute of Mathematics; Department of Information Technologies, Novosibirsk State University, beresnev@math.nsc.ru

Abstract We consider a model of competition process organized as a Stackelberg game. Two parties open their facilities with the aim to maximize profit from serving the customers which behave in accordance to a binary rule. The set of customers is unknown for the party opening its facilities first but reveals itself before the second party makes own decision. A finite set of scenarios specifying all the customer's parameters is provided to the first party, and one of the scenarios would be realized in the future. The scenarios have a known probability of realization, and the first party, beside maximizing the profit, aims to increase the probability to get a certain profit value. We formulate a problem of the first party as a bi-objective bi-level mathematical program incorporating the second party's problem at the lower-level. To approximate the set of effective solutions, we develop an ε-constraint method solving a sequence of bi-level problems with a single objective. A subset of weakly efficient solutions obtained by the method allows to analyze trade-offs caused by uncertainties in demand specification.

Keywords: *Bi-level programming, Competitive location, Multicriteria optimization, Stackelberg game*

1 Introduction

Making decisions on location of commercial facilities should involve anticipation of the competitors' reaction which is rational and prescribed by solution of some optimization problem. A competitive nature of the process can not be ignored due to the fact that an income obtaining by the facilities to be open depends on further actions of the competitors. We assume that there is a single competitor on the market or all the competitors can be aggregated into a single one. In this situation, the party locating facilities can consider itself as the Leader in a Stackelberg game. The second player of this game, called the Follower, represents the competitor who makes the decision knowing the Leader's one.

Competitive models incorporate so-called customer behavior rules determining how the customers split their demand among the facilities opened by the competitors. In the present work, a binary customer rule is utilized. It assumes that each of the customers patronizes a single facility. Choice of that facility is based on customer's preferences represented with linear order on the set of potential facility locations. In the model under consideration, a customer can be served by any facility which is more preferable than all the competitor's ones.

A demand for products or services provided by the competing parties is affected by many factors and is hardly predictable. Beside the uncertainty inherent to predicted data, the demand is sensitive to external factors influencing on its structure. We consider a situation when the Leader is provided with a finite number of possible scenarios. Each of the scenarios fully characterizes the set of customers with their attributes and has a known probability of realization. Only one of the scenarios will be realized in the future, and it is impossible to know in advance which one it would be. In the Stackelberg game framework, the Follower makes a decision after the Leader. It provides the Follower a knowledge about the decision regarding facilities opened by the Leader. Moreover, additional evidences about the demand scenario realized reveal themselves, so the Follower is able to distinguish the scenario before making own decision. Thus, unlike the Leader, the Follower is in a situation of full information and acts optimally regardless of which scenario is realized.

As like as in the maximization facility location problem, it is assumed that the parties maximize their profit calculated as an income from serving the customers after deduction of fixed costs of open facilities. The Leader itself is interested in finding not only the safest and conservative solution but more risky and profitable ones as well. To deal with it, a second objective function representing a probability to obtain a certain profit is introduced into the Leader's problem. It results in a bi-objective competitive facility location problem (BCompFLP) with the upper-level problem to maximize both the Leader's profit and the probability to get it, and a lower-level problem to maximize the Follower's profit in each of the scenarios.

2 Formalization

For brevity, let us provide a combinatorial formulation of BCompFLP instead of the one represented in terms of mathematical programming. Consider a finite set $I = \{1, \dots, m\}$ of possible locations where facilities of both the parties can be opened. It is assumed that the Leader considers a list of possible scenarios indexed by the elements of the set $S = \{1, \dots, l\}$. A finite set J_s represents the set of customers available in the case when the scenario $s \in S$ is realized.

The list of input parameters for BCompFLP consists of fixed costs f_i of opening the Leader's facility $i \in I$ and an income c_{ij} from serving the customer $j \in J_s$, $s \in S$ by the facility i. The analogical Follower's parameters would be denoted by g_i and d_{ij}, $i \in I$, $j \in J_s$, $s \in S$, respectively. The probability of the scenario $s \in S$ realization is denoted by p_s.

We assume that each customer is served by a single facility chosen with respect to preferences of the customer represented with linear order on the set I. Given i_1, $i_2 \in I$, $i_1 \neq i_2$, the relation $i_1 \succ_j i_2$ means that i_1 is more preferable for j than i_2. For a non-empty subset $I' \subseteq I$ and some $i \in I \backslash I'$, we use a notation $i \succ_j I'$ to say that $i \succ_j k$ for all $k \in I'$. Additionally, we assume that $i \succ_j \emptyset$ for all $i \in I$, $j \in J$.

Consider a non-empty subset $I_L \subseteq I$ of facilities opened by the Leader and a scenario $s \in S$. We assume that the Follower cannot open own facilities in the locations occupied by the Leader. For a subset $I_F \subseteq I \backslash I_L$ of facilities opened by the Follower, its profit $F^s(I_F)$ is calculated in the following way:

$$F^s(I_F) = -\sum_{i \in I_F} g_i + \sum_{j \in J_s} \max_{i \in I_F | i \succ_j I_L} d_{ij}, \tag{1}$$

where a maximum over the empty set is assumed to be equal to zero. Let $C_s(I_F)$ denotes the value of income obtaining by the Leader in a case when the scenario s is realized, and the set of Follower's facilities equals to I_F: $C_s(I_F) = \sum_{j \in J_s} \max_{i \in I_L | i \succ_j I_F} c_{ij}$. The Follower is rational and chooses a subset $\tilde{I}_F^s$ maximizing the profit value $F^s(\tilde{I}_F^s)$. In the pessimistic case, the subset $\tilde{I}_F^s$ is the least desired optimal solution of the Follower, i. e. such that the Leader's income function $C_s(\cdot)$ is minimized. Let $\tilde{C}_s$ denotes the value $C_s(\tilde{I}_F^s)$.

Since the scenario is chosen randomly after the Leader's decision, its profit can be expressed in a form $L(I_L) = -\sum_{i \in I_L} f_i + \xi$, where ξ is a random variable taking the value $\tilde{C}_s$ with a probability p_s. The Leader is interested in finding a subset of facilities maximizing a profit obtained. The second Leader's objective is maximizing the probability of obtaining a certain profit. Thus, the BCompFLP can be written as

$$\max_{I_L \subseteq I, C \geq 0} \left(-\sum_{i \in I_L} f_i + C \right), \tag{2}$$

$$\max_{p_0 \geq 0} p_0, \tag{3}$$

provided that $p(\xi \geq C) \geq p_0$, where $p(\cdot)$ is a probability measure. Continuous variables C and p_0 stand for the income value and the probability to obtain an income not less than C.

3 Main results

On the basis of the upper bound procedures proposed in [1], we develop a branch-and-bound algorithm obtaining an optimal solution of the Leader's problem with the second objective function (3) restricted to take values not less than a certain threshold [2]. By solving a series of subproblems with different threshold values, we get a set of weakly-efficient solutions of the Leader's problem. Depending on the application, the solutions obtained can be used to study "risks vs. profit" trade-offs, stability of Leader's solution with respect to perturbations of the input data and some other factors which are of practical importance. In numerical experiments, we show the spectrum of effects exploring by the model. Instances with twenty candidate locations can be processed by the method in a reasonable time.

References

[1] Melnikov A. and Beresnev V., *Upper Bound for the Competitive Facility Location Problem with Quantile Criterion*, In: Kochetov Y., Khachay M., Beresnev V., Nurminski E., Pardalos P. (eds) Discrete Optimization and Operations Research. DOOR 2016. Lecture Notes in Computer Science, vol 9869 (2016). Springer, Cham.

(2016) 77: 451.

[2] Ivanov, S.V. and Morozova, M.V., *Stochastic problem of competitive location of facilities with quantile criterion*, Automation and Remote Control, **77 (3)**, 451–461.

On optimizing over the efficient set

Kahina Ghazli *

Department of Operational Research , University of Sciences and Technology Houari Boumediene (USTHB), kah.ghazli@gmail.com

Nicolas Gillis

Department of Mathematics and Operational Research , University of Mons , nicolas.gillis@umons.ac.be

Mustapha Moulaï

Department of Operational Research , University of Sciences and Technology Houari Boumediene (USTHB), mmoulai@usthb.dz

Abstract Multiple-objective optimization (also referred to as vector or multi-criteria optimization) deals with the situations where one wishes to minimize several conflicting objectives. Formally, multiple-objective optimization can be formulated as follows

$$\begin{array}{lccc} \min_x & \big(f_1(x), \ldots, f_p(x)\big) \\ \text{such that} & x \in X, \end{array} \tag{V}$$

where $p \geq 2$, $x = (x_1, \ldots, x_n)$ is the vector of decision variables, $X \subseteq \mathbb{R}^n$ represent the feasible set and $f_k : \mathbb{R}^n \to \mathbb{R}$ $(k = 1, \ldots, p)$ are the objective functions.

Unlike the case with a single objective, the optimal solution for multiple-objective optimization is in general not a single point but a set of solutions where each solution represents a compromise between the different objectives. To solve this type of problems, one has to find all efficient solutions or weakly efficient solutions in the sense of the following definitions [2]. A feasible point $\hat{x} \in X$ is an efficient (Pareto optimal, nondominated) solution for problem (V) if and only if there does not exist $x \in X$ such that $f(x) \leq f(\hat{x})$ and $f(x) \neq f(\hat{x})$. It is a weakly efficient (weakly Pareto optimal, weakly nondominated) solution for problem (V) if and only if there does not exists $x \in X$ such that $f(x) < f(\hat{x})$. Let us denote X_E and X_{WE} the set of all efficient solutions and the set of all weakly efficient solutions for problem (V), respectively, so that $X_E \subseteq X_{WE}$.

In many situations, the decision-making process does not requires explicit enumeration of all efficient solutions, but only efficient solutions achieving the optimum of some scalar function expressing the decision maker's preferences within the set of Pareto optimal solutions. This is the problem of *optimizing over the efficient set*, introduced in [1]. This problem is given by

$$\begin{array}{ll} \min_x & \phi(x) \\ \text{such that} & x \in X_E \subseteq X, \end{array} \tag{PE}$$

where $\phi : \mathbb{R}^n \to \mathbb{R}$ is a real-valued function. The problem (PE) is among the difficult problems in global optimization because the efficient set X_E is not convex in general, even in the linear case where the objective functions f_k's are linear and the feasible set X is a polyhedron.

Most of the proposed methods to tackle this problem focus on the linear case. Several methods were developed following the work of [1]; see for example the survey [3] and the references therein. However, the literature dealing with this problem is relatively poor for nonlinear multiple objective problems due to its difficulty. Most of the algorithms proposed in the literature seek to globally solve this problem through global optimization techniques. It is worth noting that such techniques are only able to solve moderate sized problems. In this paper, we propose a numerical method to tackle this problem when $\phi(x)$ is a convex function, the objective functions f_k, $k = 1, \ldots, p$, are quadratic convex and the feasible set X is nonempty, compact and convex. This algorithm is based on a penalty approach, using a sequence of convex nonlinear subproblems that can be solved efficiently. Our idea to tackle this problem is as follows. The problem (PE) is reformulated as a nonlinear bilevel programming problem (NBLP) based on the characterization of the efficient solutions by means of scalar optimization problem derived from the multiple-objective problem, and then to construct a penalty function that is a combination of the objective functions of the upper and the lower levels of the equivalent bilevel reformulation (NBLP). The resulting penalty problem is then solved by optimizing alternatively over the lower level variables and the upper level variables.

The proposed algorithm is shown to perform well on a set of standard problems from the literature, as it allows to obtain optimal solutions in all cases. In addition, it can be adapted easily to obtain a weakly efficient solution.

Keywords: *multiple objective nonlinear optimization, (weakly) efficient set, penalty approach*

References

[1] Phillip, J., *Algorithms for the Vector Maximization Problem*, Mathematical Programming **2** (1972), 207–229.

[2] Ehrgott, M., "Multicriteria Optimization," Springer-Verlag **2**, Berlin Heidelberg, 2005.

[3] Yamamoto, Y., *Optimization over the efficient set: overview*, Journal of Global Optimization **22** (2003), 285–317.

Contributed session

Dealing with the leader's decision risk in semivectorial bilevel programming problems

Maria João Alves*
CeBER and Faculty of Economics of University of Coimbra / INESC Coimbra, Portugal , `mjalves@fe.uc.pt`

Carlos Henggeler Antunes
Dept. of Electrical and Computer Engineering of University of Coimbra / INESC Coimbra, Portugal, `ch@deec.uc.pt`

Abstract The *semivectorial bilevel programming* problem (SVBLP) refers to bilevel problems comprising a single objective at the upper-level and multiple objectives at the lower-level. In SVBLP, the lower-level problem has a number of tradeoff solutions (efficient solutions) for each upper-level solution. Thus, in addition to the theoretical and computational difficulties in computing solutions to these problems, further issues arise in the decision process resulting from the fact that the leader (upper-level decision maker) does not know a-priori which efficient solution the follower (lower-level decision maker) will choose according to his (unknown) preferences. Therefore, the complexities of these problems are beyond those already inherent to conventional multiobjective programming and bilevel programming.

Consider the following formulation for the SVBLP:

$$\begin{aligned} \min_{x} \quad & F(x,y) \\ s.t. \quad & G(x) \leq 0 \\ & y \in \arg\min_{y} \{f_1(x,y), \cdots, f_m(x,y) : g(x,y) \leq 0\} \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^{n_1}$ and $y \in \mathbb{R}^{n_2}$ denote the upper-level and the lower-level variables, respectively, and m is the number of objective functions at the lower-level. We assume that the set $X = \{x \in \mathbb{R}^{n_1} : G(x) \leq 0\}$ is nonempty and compact and, for each $x \in X$, the set $Y(x) = \{y \in \mathbb{R}^{n_2} : g(x,y) \leq 0\}$ is also nonempty and compact. For each $x \in X$ there is a set $\Psi_{ef}(x)$ of efficient (Pareto optimal or nondominated) solutions to the lower-level problem. $\Psi_{ef}(x) = \{y \in Y : \text{there is no } y' \in Y(x) | f(x,y') \prec f(x,y)\}$ where $\prec$ denotes the dominance relation, i.e., $f(x,y') \prec f(x,y)$ iff $f_j(x,y') \leq f_j(x,y)$ for all $j = 1,\cdots,m$, and $f_j(x,y') < f_j(x,y)$ for at least one j. Generally $\Psi_{ef}(x)$ is not singleton and, thus, the optimal solution to problem (1) may be ambiguous. The two main ways to deal with the problem suggested in the literature are the optimistic and the pessimistic approaches, which lead to two reformulations of (1).

In the optimistic approach the leader assumes that the follower is willing to support him and selects the solution among $\Psi_{ef}(x)$ that is the best for the leader. Thus, an *optimistic* solution (x^o, y^o) is a solution that optimizes the optimistic formulation: $\min_{x,y} F(x,y)$ s.t. $G(x) \leq 0, y \in \Psi_{ef}(x)$.

In the pessimistic approach the leader prepares for the worst case. The leader chooses the x that leads to a solution with minimum F in view of the follower's decisions y worst for the leader. The *pessimistic* solution (x^p, y^p), if it exists, is an optimal solution to the pessimistic formulation: $\min_x \max_y F(x, y)$ s.t. $G(x) \leq 0, y \in \Psi_{ef}(x)$.

Most theoretical and algorithmic contributions made thus far to solve SVBLP have generally adopted an optimistic formulation. Several solution approaches (exact and heuristic) have been proposed to compute or approximate the optimistic solution, especially for problems with multiobjective linear problems at the lower-level, while few works have addressed the pessimistic formulation.

However, accepting an optimistic approach means assuming that the follower is indifferent to any of his efficient solutions, i.e. he is indifferent between optimizing any one of his objective functions or even any combination of them. Therefore, the larger the number of lower level objective functions is, more freedom is given by the optimistic approach to the leader for choosing the solution more convenient to him. In practice this assumption is seldom realistic. Thus, it is important to acknowledge the risk the leader takes if he adopts an optimistic procedure to a real SVBLP. On the other hand, the pessimistic solution may be very conservative by just focusing on the worst choice of the follower in view of the leader's interests.

In addition to the optimistic and pessimistic solutions, there are other types of solutions that can also provide useful information to the leader about the risk he is running into when making a specific decision. In particular, the following solutions may be relevant for decision support purposes: the result of a failed optimistic approach – *deceiving* solution – and, on the other hand, the solution resulting from a successful pessimistic approach – *rewarding* solution.

The deceiving solution is the outcome when the leader chooses x according to the optimistic approach (believing that the follower will pursue his interests) but the follower does not react accordingly and takes the decision with the worst value for the leader's objective function. Thus, given the optimistic upper level decision x^o, the *deceiving* solution is $(x^d, y^d) = (x^o, y^d)$ where y^d is given by $\max_y F(x^o, y)$ s.t. $y \in \Psi_{ef}(x^o)$. On the other hand, if a pessimistic approach is pursued by the leader but the follower's reaction is the most favorable to the leader, a rewarding solution is obtained. Thus, given the pessimistic upper level decision x^p, the *rewarding* solution is $(x^r, y^r) = (x^p, y^r)$ such that y^r is given by $\min_y F(x^p, y)$ s.t. $y \in \Psi_{ef}(x^p)$. These four *extreme* solutions – *optimistic*, *pessimistic*, *deceiving* and *rewarding* – give important insights to the leader of possible outcomes and ranges of values resulting from a risky or a conservative approach.

Let us call optimistic efficient frontier (O) the set comprising the solutions (x, y') such that $x \in X$ and $y' \in O(x) = \left\{ \arg\min_y \{F(x, y) : y \in \Psi_{ef}(x)\} \right\}$, and pessimistic efficient frontier (P) the set comprising the solutions (x, y'') such that $x \in X$ and $y'' \in P(x) = \left\{ \arg\max_y \{F(x, y) : y \in \Psi_{ef}(x)\} \right\}$. Fig.1(a) illustrates the F values in the optimistic and pessimistic efficient frontiers of a SVBLP linear problem with one upper level variable (x). In this example there is a significant difference between the optimistic and the deceiving solutions for the leader's objective function F. Therefore, if the leader adopts for an optimistic approach he takes a high risk, since the deceiving solution is very bad. Conversely, there is a small difference between the pessimistic F and the rewarding one, being the F value in the rewarding solution close to the optimistic F.

In addition to these four *extreme* solutions, we introduce another solution concept, the *moderate* solution, which provides the leader the highest expected value considering an index

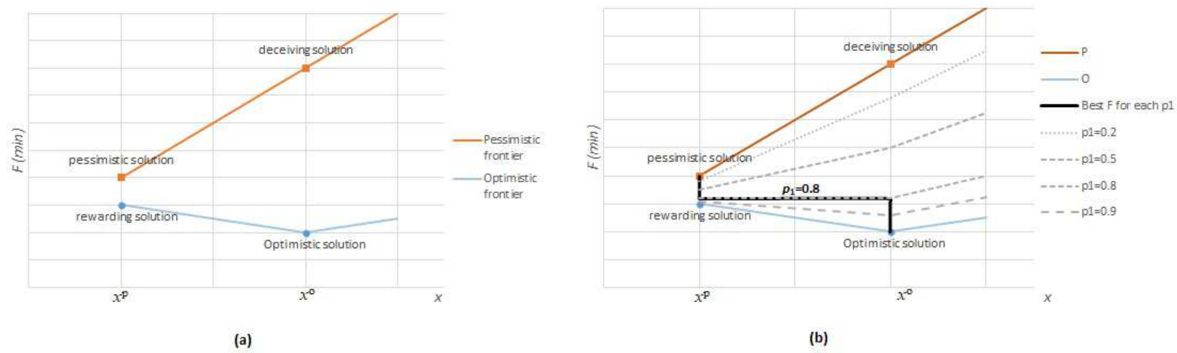


Figure 1: Example of F values in O and P frontiers, and for different optimism p_1 values

of optimism vs. pessimism. Consider $p_1 \in [0, 1]$ an optimism index defined by the leader (which may represent the probability perceived by leader for the follower choosing, for each x, the solution $y \in \Psi_{ef}(x)$ that is the best for the leader) and $p_2 \in [0, 1]$, $p_2 = 1 - p_1$ the pessimism index (probability for the follower choosing, for each x, the solution $y \in \Psi_{ef}(x)$ that is the worst for the leader). The solution that optimizes the expected value for the leader's objective function is given by:

$$\begin{aligned} \min_{x} \quad & F(x, y) \\ s.t. \quad & G(x) \leq 0 \\ & F(x, y) \geq p_1 F(x, y') + p_2 F(x, y'') \\ & y' \in O(x), y'' \in P(x) \end{aligned} \tag{2}$$

Fig. 1(b) shows, for the example presented in Fig. 1(a), the lines of F values for different values of p_1. In this example, for values of the optimism index p_1 from 1 to 0.8, the best leader's decision x according to (2) is the optimistic one, x^o; for $p_1 = 0.8$, the F expected value is equal for the leader's decisions x^o and x^p; for $p_1 < 0.8$, the best leader's decision x according to (2) is the pessimistic one, x^p. This information complements the analysis above of the four extreme solutions concerning the assessment of the leader's risk in each decision.

To summarize, the existence of multiple objective functions at the lower-level of a bilevel problem poses additional difficulties for the leader to anticipate the follower's reaction to the underlying tradeoffs between his multiple objectives. Therefore, different types of solutions can provide broader information to the leader. The optimistic, pessimistic, deceiving and rewarding solutions characterize distinct attitudes of the leader and the follower, delimiting the ranges of possible optimal values for the leader taking into account the follower's extreme decisions. In order to provide further decision support to the leader in SVBLP, other solutions may be computed (moderate solutions) that optimize the expected value for the leader's objective function considering different probabilities of the follower's decision being in favor or against the interests of the leader.

In this presentation we will introduce these different types of solutions to SVBLP, illustrating their usefulness using several examples, with linear and non-linear functions.

Keywords: *semivectorial bilevel programming, multiobjective optimization, pessimistic vs. optimistic approaches, risk*

Valid Inequalities for Mixed Integer Bilevel Linear Optimization Problems

Ted K. Ralphs*
Department of Industrial and Systems Engineering, Lehigh University, ted@lehigh.edu
Suresh Bolusani
Department of Industrial and Systems Engineering, Lehigh University, bsuresh@lehigh.edu
Sahar Tahernejad
Department of Industrial and Systems Engineering, Lehigh University, sat214@lehigh.edu

Abstract

We provide an overview of the known classes of inequalities valid for the feasible region of a *mixed integer bilevel linear optimization problem* (MIBLP). The overview includes a summary of both the theoretical basis for the inequalities and an analysis of their performance within the MibS open source solver [1]. We further describe an algorithm for generating inequalities that support the convex hull of the feasible region and use this procedure as a baseline for comparing the strength of various other classes.

We let $x \in X$ and $y \in Y$ represent the set of variables controlled by the first- and second-level *decision makers* (DMs) respectively, where $X = \mathbb{Z}_+^{r_1} \times \mathbb{R}_+^{n_1 - r_1}$ and $Y = \mathbb{Z}_+^{r_2} \times \mathbb{R}_+^{n_2 - r_2}$. The general form of an MIBLP is then given by

$$\min \left\{ cx + d^1 y \;\middle|\; x \in X, y \in \mathcal{P}_1(x) \cap \mathcal{P}_2(x) \cap Y, d^2 y \le \phi(b^2 - A^2 x) \right\}, \tag{MIBLP}$$

where

$$\mathcal{P}_1(x) = \left\{ y \in \mathbb{R}_+^{n_2} \;\middle|\; G^1 y \ge b^1 - A^1 x \right\} \text{ and}$$
$$\mathcal{P}_2(x) = \left\{ y \in \mathbb{R}_+^{n_2} \;\middle|\; G^2 y \ge b^2 - A^2 x \right\}$$

denote the set of the points satisfying the linear constraints of the first- and second-level problems for a given $x \in \mathbb{R}^{n_1}$, respectively and

$$\phi(\beta) = \min \left\{ d^2 y \;\middle|\; G^2 y \ge \beta, y \in Y \right\} \quad \forall \beta \in \mathbb{R}^{m_2}. \tag{VF}$$

With respect to a given $x \in X$, the *rational reaction set* is defined as

$$\mathcal{R}(x) = \operatorname{argmin} \left\{ d^2 y \;\middle|\; y \in \mathcal{P}_2(x) \cap Y \right\}$$

and contains $y \in Y$ such that y is an optimal solution to the second-level problem parameterized on x. The *bilevel feasible region* is defined as

$$\mathcal{F} = \left\{ (x, y) \in X \times Y \mid y \in \mathcal{P}_1(x) \cap \mathcal{R}(x) \right\}.$$

Dropping the optimality constraint with respect to the second-level problem, we obtain the relaxed feasible region

$$\mathcal{S} = \left\{ (x, y) \in X \times Y \mid y \in \mathcal{P}_1(x) \cap \mathcal{P}_2(x) \right\}.$$

Further dropping the integrality constraints, we get

$$\mathcal{P} = \left\{(x, y) \in \mathbb{R}^{n_1 \times n_2}_{+} \;\middle|\; y \in \mathcal{P}_1(x) \cap \mathcal{P}_2(x)\right\}.$$

We have $\mathcal{F} \subseteq \mathcal{S} \subseteq \mathcal{P}$.

We assume that all first-level variables with at least one non-zero coefficient in the second-level problem (so-called *linking variables*) are integer-valued, i.e.,

$$L = \left\{i \in \{1, \ldots, n_1\} \;\middle|\; A^2_i \neq 0\right\} \subseteq \{1, ..., r_1\},$$

where A^2_i represents the i^{th} column of matrix A^2. This assumption guarantees that the optimal value of (MIBLP) is attainable (when the feasible region is nonempty). It is easily shown that after fixing the values of linking variables, the MIBLP is equivalent to a *mixed integer linear optimization problem* (MILP).

Proposition 1 *Let $\bar{\mathcal{F}} \subseteq \{(x, y) \in \mathcal{F} \mid x_L = \gamma\}$ for some $\gamma \in \mathbb{Z}^L$, then*

$$\min_{(x,y)\in\bar{\mathcal{F}}} cx + d^1 y = \min\left\{cx + d^1 y \;\middle|\; (x, y) \in \mathcal{S}, d^2 y \leq \phi(b^2 - A^2 x), x_L = \gamma\right\}. \qquad (UB)$$

As usual, a valid inequality for a given optimization problem is one that is satisfied by all feasible solutions to the problem ($\mathcal{F}$ in this case). In the notation of (MIBLP), we have the following.

Definition 1 (Valid Inequality) $(\alpha^x, \alpha^y, \beta) \in \mathbb{R}^{n_1+n_2+1}$ *is a valid inequality for $\mathcal{F}$ if*

$$\mathcal{F} \subseteq \left\{(x, y) \in \mathbb{R}^{n_1+n_2} \;\middle|\; \alpha^x x + \alpha^y y \geq \beta\right\}.$$

Generally speaking, the goal of imposing valid inequalities during a branch-and-cut algorithm is to remove areas of the relaxed feasible region which do not belong to $\mathcal{F}$. However, there are times when we allow the removal of solutions that are feasible, but that we can show are *non-improving.*

Definition 2 (Valid Improving Inequality) $(\alpha^x, \alpha^y, \beta) \in \mathbb{R}^{n_1+n_2+1}$ *is a valid improving inequality for $\mathcal{F}$ if*

$$\operatorname{conv}\left(\left\{(x, y) \in \mathcal{F} \;\middle|\; c^1 x + d^1 y < U\right\}\right) \subseteq \left\{(x, y) \in \mathbb{R}^{n_1+n_2} \;\middle|\; \alpha^x x + \alpha^y y \geq \beta\right\}$$

where U represents the best known upper-bound for (MIBLP)*.*

In the remainder of abstract, when we say "valid inequality," we are referring to "valid improving inequality".

In the context of MIBLPs, the main aim of generating a valid inequality is to remove either

1. An extreme point $(\bar{x}, \bar{y})$ of $\mathcal{P}$ (or the feasible region of the current relaxation), where $(\bar{x}, \bar{y}) \notin X \times Y$.
2. An extreme point $(\bar{x}, \bar{y})$ of $\mathcal{P}$ (or the feasible region of the current relaxation) such that $(\bar{x}, \bar{y}) \in X \times Y$, but $\bar{y} \notin \mathcal{R}(\bar{x})$.
3. All $(x, y) \in \mathcal{P}$ such that $x \in \operatorname{proj}_x(\mathcal{F})$ and $x_L = \lambda \in \mathbb{Z}^L$.

The Cases 1 and 2 above occur when the extreme point $(\bar{x}, \bar{y})$ resulting from solving the relaxation is not bilevel feasible. Note that, unlike in the case of the feasible region of an MILP, the feasible region of the relaxation has integral extreme points that are nevertheless infeasible (Case 2). Case 3 arises anytime we solve the problem (UB), since all $(x, y) \in \mathcal{F}$ with $x_L = \lambda \in \mathbb{Z}^L$ can be removed, as they are non-improving by construction.

We consider inequalities of three classes, based on what types of solutions they remove.

1. Feasibility cuts: Inequalities that are violated by a fractional extreme point of $\mathcal{P}$ (as in Case 1), but are valid for $\mathcal{S}$.

2. Optimality cuts: Inequalities that are violated by an integral extreme point of $\mathcal{P}$ (as in Case 2 above), but are valid for $\mathcal{F}$.

3. Projected optimality cuts: Inequalities violated by non-improving bilevel feasible solutions (as in Case 3), but valid for $\operatorname{conv}\left(\left\{(x, y) \in \mathcal{F} \;\middle|\; cx + d^1 y < U\right\}\right)$.

We describe inequalities in each of these classes and compare their performance based on their impact on the branch-and-cut algorithm described in [1], as well as on comparison to the aforementioned method for generating strong supporting inequalities.

Keywords: *valid inequality, valid improving inequality, mixed integer bilevel linear optimization*

References

[1] Sahar Tahernejad, Ted K. Ralphs, and Scott T. DeNegre, "A Branch-and-Cut Algorithm for Mixed Integer Bilevel Linear Optimization Problems and its Implementation," COR@L Technical Report **16T-015-R3**, 2017.

Contributed session

A Global Solution Approach for Mixed-Integer Quadratic Bilevel Problems

Thomas Kleinert*
Chair of EDOM, FAU Erlangen-Nürnberg, thomas.kleinert@fau.de

Martin Schmidt
Chair of EDOM, FAU Erlangen-Nürnberg, mar.schmidt@fau.de

Abstract Bilevel, or multilevel problems in general, become increasingly popular in applied mathematical optimization. In economics, for instance, the modeling of energy markets leads quite naturally to multilevel models due to the presence of several market participants with hierarchic decisions and opposing objectives. Other real-life applications arise, e.g., in the fields of gas network optimization, critical infrastructure defense, and machine learning. Even in their simplest form, i.e., when the upper and lower level both are continuous and linear, bilevel problems are NP-hard. The more general case of mixed-integer linear bilevel problems (MILBP) is Σ_p^2-hard. However, a couple of promising approaches for MILBPs with various properties have been published in recent years.

A well-known technique for linear bilevel problems with a convex lower level is the reformulation of the lower level by its KKT conditions. The resulting single-level problem can then be solved, e.g., by branching on the KKT complementarity conditions, or by a big-M reformulation of the KKT complementarity conditions. Another option is to use strong duality of convex optimization. The idea is to extend the convex lower-level problem by its dual problem and a linear equality constraint that enforces strong duality; see, e.g., [1]. In general, both approaches contain products of primal first-level and dual second-level variables. This may not be a problem for integer linking variables. However, in case of continuous linking variables, there may arise problems of linearizing these products. Furthermore, there is hardly any work applying these techniques to problems with a nonlinear lower-level objective function.

Very recently, several authors proposed general-purpose branch-and-cut techniques for linear bilevel problems with mixed-integer aspects on both levels. The techniques evolved from problems, where the upper level does not contain any lower-level variables and problems with integer linking variables only, to the general case with variables of both levels in the first and second level and without restrictions to linking variables; see, e.g., [2]. However, all of these approaches "only" deal with linear objective functions.

When analyzing problems from the field of energy market design, one is often faced with bilevel problems with a convex lower level and integer linking variables but with quadratic objectives on both levels. In this case, except of the KKT reformulation, none of the stated approaches can be used in a straightforward manner. In fact, to the best of our knowledge, no general-purpose solution techniques for mixed-integer quadratic bilevel problems exist. In [3], two problem-specific solution techniques for a mixed-integer quadratic trilevel problem with convex lower levels are presented. The first one uses the special structure of the model in

order to reduce the problem to a bilevel problem and then applies a standard KKT reformulation with big-M linearizations of the KKT complementarity conditions. This approach ends up with a single-level mixed-integer quadratic program (MIQP) but has shown to be very challenging from a numerical point of view. It is significantly outperformed by the second proposed technique, a generalized Benders decomposition. The latter approach, however, is only applicable, when lower-level variables are not present in the respective upper level constraints.

In order to tackle this drawback, we propose a novel solution approach for quadratic bilevel problems with integer linking variables and convex lower-level problems. In particular, we deal with mixed-integer quadratic upper level problems of the form

$$\min_{x_1,y_1} \quad f_1(x_1,y_1,y_2) = \frac{1}{2}(x_1^\top, y_1^\top)P_1\begin{pmatrix}x_1\\ y_1\end{pmatrix} + c_1^\top\begin{pmatrix}x_1\\ y_1\end{pmatrix} + \frac{1}{2}y_2^\top Q_1 y_2 + d_1^\top y_2 \tag{1a}$$

$$\text{s.t.} \quad A_1x_1 + B_1y_1 + C_1y_2 \le b_1\,, \tag{1b}$$

$$x_1 \in \mathbb{Z}^{n_1},\; y_1 \in \mathbb{R}^{m_1}, \tag{1c}$$

where P_1 and Q_1 are positive definite matrices, and convex-quadratic lower-level problems of the form

$$\min_{y_2} \quad f_2(x_1,y_1,y_2) = \frac{1}{2}(x_1^\top, y_1^\top)P_2\begin{pmatrix}x_1\\ y_1\end{pmatrix} + c_2^\top\begin{pmatrix}x_1\\ y_1\end{pmatrix} + \frac{1}{2}y_2^\top Q_2 y_2 + d_2^\top y_2 \tag{2a}$$

$$\text{s.t.} \quad A_2x_1 + C_2y_2 \le b_2\,, \tag{2b}$$

$$y_2 \in \mathbb{R}^{m_2}\,, \tag{2c}$$

with positive definite matrices P_2 and Q_2.

Our approach builds on strong duality of convex optimization. In contrast to linear problems, this involves handling quadratic constraints in case of quadratic problems. For the lower-level problem (2), its dual problem is given by

$$\max \quad g(\lambda) = -\frac{1}{2}\lambda c_2 Q_2^{-1} c_2^\top \lambda + (A_2x_1 - b_2)^\top\lambda - \frac{1}{2}d_2^\top Q_2^{-1} d_2 + \frac{1}{2}y_1^\top P_2 y_1 + c_2^\top y_1 \tag{3a}$$

$$\text{s.t.} \quad \lambda \ge 0\,, \tag{3b}$$

which is a concave problem. We can now replace the lower level (2) by

$$\text{primal feasibility: } (2b), (2c), \tag{4a}$$

$$\text{dual feasibility: } (3b), \tag{4b}$$

$$\text{strong duality: } F(x_1,y_1,y_2,\lambda) = f_2(x_1,y_1,y_2) - g(\lambda) \le 0\,. \tag{4c}$$

We note that (4c) is a quadratic but convex constraint. Hence, its first-order Taylor approximation at a point $(\tilde{x}_1, \tilde{y}_1, \tilde{y}_2, \tilde{\lambda})$ provides an underestimator, i.e.,

$$F(\tilde{x}_1,\tilde{y}_1,\tilde{y}_2,\tilde{\lambda}) - \nabla_{y_2,\lambda}F(\tilde{x}_1,\tilde{y}_1,\tilde{y}_2,\tilde{\lambda})\begin{pmatrix}y_2 - \tilde{y}_2\\ \lambda - \tilde{\lambda}_2\end{pmatrix} \le F(x_1,x_2,y_2,\lambda)\,. \tag{5}$$

This can be exploited by an outer approximation scheme that relaxes the optimality of the lower level in the first place. Optimality is then iteratively restored by adding outer approximation cuts of the form

$$F(\tilde{x}_1,\tilde{y}_1,\tilde{y}_2,\tilde{\lambda}) - \nabla_{y_2,\lambda}F(\tilde{x}_1,\tilde{y}_1,\tilde{y}_2,\tilde{\lambda})\begin{pmatrix}y_2 - \tilde{y}_2\\ \lambda - \tilde{\lambda}_2\end{pmatrix} \le 0\,, \tag{6}$$

where $(\tilde{x}_1, \tilde{y}_1, \tilde{y}_2, \tilde{\lambda})$ denotes the solution of the previous iteration. Following this approach, every iteration requires to solve an MIQP of the form

$$\begin{aligned} \min \quad & f_1(x_1, y_1, y_2) && (7a) \\ \text{s.t.} \quad & (1b), (1c), (2b), (2c), (3b), && (7b) \\ & \text{cuts of the form } (6)\,. && (7c) \end{aligned}$$

Hence, our approach reduces the bilevel problem to a series of single-level MIQPs that can be solved using powerful and sophisticated MIQP-techniques. We provide a termination criterion and prove that this solution technique is correct under mild assumptions. In a first preliminary numerical test on a small test bed, the stated approach showed promising results. In particular, it indicates better runtimes compared to the well-known standard KKT reformulation. However, for a definitive judgment, a detailed computational study has to be conducted.

Keywords: *Quadratic Bilevel Programming, Mixed-Integer Optimization, Outer Approximation*

References

[1] Gabriel, S., Conejo, A., Fuller, J., Hobbs, B. and Ruiz, C. *Complementarity modeling in energy markets*, Springer Science & Business Media, Vol. 180, 2012.

[2] Fischetti, M., Ljubić, I., Monaci, M., and Sinnl, M., *A new general-purpose algorithm for mixed-integer bilevel linear programs*, Operations Research, Vol. 65-6, 2017.

[3] Grimm, V., Kleinert, T., Liers, F., Schmidt, M. and Zöttl, G., *Optimal price zones of electricity markets: A mixed-integer multilevel model and global solution approaches*, Optimization Methods and Software, 2017.

Tropical geometry and discrete convexity applied to bilevel programming

Eytard Jean-Bernard *
INRIA, CMAP, Ecole Polytechnique, CNRS, jean-bernard.eytard@inria.fr

Akian Marianne
INRIA, CMAP, Ecole Polytechnique, CNRS, marianne.akian@inria.fr

Bouhtou Mustapha
Orange Labs, mustapha.bouhtou@orange.com

Gaubert Stéphane
INRIA, CMAP, Ecole Polytechnique, CNRS, stephane.gaubert@inria.fr

Koshevoy Gleb
CEMI, Russian Academy of Science, koshevoy@cemi.rssi.ru

Abstract We study special bilevel programming problems, modelling different economic issues. We develop an approach based on tropical geometry. We identify a class of bilevel problems which can be solved in polynomial time thanks to properties of discrete convexity. This applies in particular to a congestion problem in mobile data networks, proposed by Orange. Finally, we show that this class of bilevel problems can be obtained as a limit of competitive equilibria for indivisible goods.

Keywords: *Bilevel programming, tropical geometry, discrete convexity, competitive equilibria*

1 A class of bilevel problems

We consider the following optimistic bilevel problem :

$$\min_{y\in\mathbb{R}^n} f(C^T x^*, y) \quad \text{s.t.} \quad x^* \in \arg\max_{x\in\mathcal{P}} \langle \rho + Cy, x\rangle \tag{1}$$

where $\mathcal{P}$ denotes a polytope of $\mathbb{R}^k$, $\rho \in \mathbb{R}^k$ and $C \in \mathcal{M}_{k,n}(\mathbb{R})$, and a combinatorial version:

$$\min_{y\in\mathbb{R}^n} f(C^T x^*, y) \quad \text{s.t.} \quad x^* \in \arg\max_{x\in\mathcal{I}(\mathcal{P})} \langle \rho + Cy, x\rangle \tag{2}$$

where $\mathcal{I}(\mathcal{P})$ denotes the integer points of $\mathcal{P}$ ($\mathcal{I}(\mathcal{P}) = \mathcal{P} \cap \mathbb{Z}^k$).

We propose here an approach based on tropical geometry. We consider the max-plus semifield $\mathbb{R} \cup \{-\infty\}$ with laws $\oplus$ et $\odot$ defined by $a \oplus b = \max(a, b)$ and $a \odot b = a + b$.

By analogy with classical polynomials, we define a *tropical polynomial* by $P(x) = \bigoplus_i c_i \odot x^{\odot a_i} = \max_i [c_i + \langle x, a_i\rangle]$ (here, $x^{\odot a_i}$ denotes $x_1^{\odot a_i(1)} \odot \cdots \odot x_n^{\odot a_i(n)}$). The tropical polynomials correspond to the sets of convex functions, piecewise linear with integer slopes. Each part

$c_i + \langle x, a_i \rangle$ is a *tropical monomial.* We denote by *tropical hypersurface* associated with the tropical polynomial P the set of points where P is not differentiable, that is the points where the maximum is "attained" at least twice.

In the Problem 1, the low-level corresponds to the evaluation of a tropical polynomial in y:

$$\max_{x\in\mathcal{P}} \langle \rho + Cy, x\rangle = \max_{x\in\mathcal{P}} \left[\langle y, C^T x\rangle + \langle \rho, x\rangle\right] = \max_{z\in C^T\mathcal{P}} \left[\langle y, z\rangle + \varphi(z)\right] = \bigoplus_{z\in C^T\mathcal{P}} \varphi(z) \odot y^{\odot z} = P(y)$$

where the function φ is defined on $\mathbb{R}^n$ by $\varphi(z) = \max_{x\in\mathcal{P},\ z=C^T x}\langle \rho, x\rangle$ (the function φ is only finite on the polyhedron $C^T\mathcal{P}$). We show that the tropical polynomial P induces a subdvision $\mathcal{S}$ of $\mathbb{R}^n$, in which each cell $\mathcal{C} \subset \mathcal{S}$ corresponds to the set of vectors y for which the same group of monomials realizes the maximum is P. For each cell $\mathcal{C}$, we denote by $\Delta(\mathcal{C})$ the sets of degrees z^* of the maximal monomial. Consider for example $P(y) = \max(y_1, y_2, 0)$ with $y \in \mathbb{R}^2$. The first monomial is maximal on the domain $\mathcal{C}_1 = \{y \in \mathbb{R}^2 \mid y_1 \geq y_2,\ y_1 \geq 0\}$. The degree of this maximal monomial is $(1, 0)$. Hence, we have $\Delta(\mathcal{C}_1) = \{(1, 0)\}$. The second monomial is maximal on $\mathcal{C}_2 = \{y \in \mathbb{R}^2 \mid y_2 \geq y_1,\ y_2 \geq 0\}$ ($\Delta(\mathcal{C}_2) = \{(0, 1)\}$). The intersection $\mathcal{C}_1 \cap \mathcal{C}_2$ defines another cell in which all linear combination of $(1, 0)$ and $(0, 1)$ is the degree of a maximal monomial ($\Delta(\mathcal{C}_1 \cap \mathcal{C}_2) = \text{Conv}\ ((1, 0); (0, 1))$).

The bilevel problem 1 can be rewritten as:

$$\min_{y\in\mathbb{R}^n} f(z^*, y) \quad \text{s.c.} \quad z^* \in \arg\max_{z\in C^T\mathcal{P}} \left[\langle y, z\rangle + \varphi(z)\right]$$

By subdivising $\mathbb{R}^n$ by $\mathcal{S}$, we obtain a new approach by enumerating the cells for solving the bilevel problem 1:

Theorem 1. *The bilevel problem 1 is equivalent to* $\min_{\mathcal{C}\in\mathcal{S}} \min_{y\in\mathcal{C},\ z^*\in\Delta(\mathcal{C})} f(z^*, y)$

We study a particular case in which the function f does not depend on the price vector y. According to theorem 1, the bilevel problem consists in minimizing f over the sets of degrees of the monomials which can be maximal in P. We show that this set is exactly $C^T\mathcal{P}$ for Problem 1 and $C^T\mathcal{P} \cap \mathbb{Z}^n$ for Problem 2 when, by defining $\mathcal{P}$ as $\mathcal{P} = \{x \in \mathbb{R}^k \mid Ax \leq b\}$, the matrix $\left[C\ A^T\right]$ is totally unimodular. It leads to the following decomposition theorem.

Theorem 2 (Decomposition)**.** *A solution $y^* \in \mathbb{R}^n$ of Problem 1 (respectively of Problem 2) can be obtained by:*

- *calculating $z^* \in \arg\min_{z\in C^T\mathcal{P}} f(z)$ (resp. $z^* \in \arg\min_{z\in C^T\mathcal{P}\cap\mathbb{Z}^n} f(z)$),*
- *finding $y^* \in \mathbb{R}^n$ and $x^* \in \mathcal{P}$ (resp. $x^* \in \mathcal{I}(\mathcal{P})$) such that $z^* = C^T x^*$ and $\langle \rho + Cy^*, x^*\rangle = \max_{x\in\mathcal{P}}\langle \rho + Cy^*, x\rangle$ (resp. $\langle \rho + Cy^*, x^*\rangle = \max_{x\in\mathcal{I}(\mathcal{P})}\langle \rho + Cy^*, x\rangle$).*

2 Application to a congestion problem in mobile data networks

We consider a particular case in which $k = qn$ (with $q \in \mathbb{N}$), $\mathcal{P} = \mathcal{P}_1 \times \cdots \times \mathcal{P}_q$ (we suppose that for all j, $\mathcal{P}_j \subset \mathbb{R}^n$) and $C^T = [I_q \ldots I_q]$. In this paricular case, the low-level problem can be separated in q different optimization problems, and we show that the Problem 2 becomes:

$$\min_{y\in\mathbb{R}^n} f(N) \quad \text{s.t.} \quad N = \sum_{j=1}^{q} x_j^*; \quad \forall 1 \leq j \leq q,\ x_j^* \in \arg\max_{x_j\in\mathcal{I}(\mathcal{P}_j)} \langle \rho_j + y, x_j\rangle \tag{3}$$

It corresponds to a purified version of a congestion problem in mobile data networks ([1]). A telecommunication operator proposed discounts $y \in \mathbb{R}^n$ at different times and in different cells. Each customer j determines his optimal consumption x_j^* by maximizing his utility. The operator observes the traffic $N = \sum_j x_j^*$ and propose discounts in order to balance the network, that is to minimize a congestion measure. The decomposition theorem gives a method to solve the problem, from which we can deduce exact algorithms. We show that it leads to polynomial time algorithms when the different polytopes $\mathcal{P}_j$ correspond to generalized polymatroids, and the function f is $M^\natural$-convex in sense of Murota ([2]).

3 Comparison with a competitive equilibrium problem

We compare this congestion problem with a competitive equilibrium problem for indivisible goods, introduced by Danilov, Koshevoy and Murota ([3]). A set of producers want to sell n different goods to q different customers. Each customer $j = 1, \dots, q$ wishes to buy a pack of different goods, each pack corresponding to an integer point of a generalized polymatroid. When the price vectors of the different products is p (p_i is the price of the good i), the customer j chooses the pack x_j^* which maximizes his utility. He solves $\max_{x_j \in \mathcal{I}(\mathcal{P}_j)} \langle \rho_j - p, x_j \rangle$. The producers desire to sell all their goods. They determine their optimal consumption $N^* \in \sum_j \mathcal{I}(\mathcal{P}_j)$ by maximizing their own profit. They solve $\max_{N \in \sum_j \mathcal{I}(\mathcal{P}_j)} f(N) + \langle p, N \rangle$ where f is supposed to be $M^\natural$-concave. Danilov et al. ([3]) show that there always exists a price vector $p \in \mathbb{R}^n$ such that $N^* = \sum_k x_k^*$. Moreover, N^* is obtained as a solution of the optimization problem $\max_{N \in \sum_k \mathcal{I}(\mathcal{P}_j)} f(N) + \psi(N)$ where ψ denotes the supremal convolution of the functions $x_j \mapsto \langle \rho_j, x_j \rangle$. It corresponds to the maximization of the sum of three $M^\natural$-concave functions. Under certain conditions (f separable concave for example), it can be reduced to the maximization of the sum of two $M^\natural$-concave functions, which can be solved in polynomial time.

We modify this competitive equilibrium problem by introducing a parameter $\lambda \geq 0$, such that the optimization problem of the producers becomes $\max_{N \in \sum_k \mathcal{I}(\mathcal{P}_k)} f(N) + \lambda \langle p, N \rangle$. According to the decomposition theorem, we show that the case $\lambda = 0$ is equivalent to the congestion problem (3) (avec $p = -y$). Morevoer, the problem (3) corresponds to a "limit" case of the competitive equilibrium problem when λ goes to 0. We prove that there exists a critical value λ_C such that for all $0 < \lambda < \lambda_C$, the competitive equilibria correspond to an optimal N of the Problem (3). Thanks to this approach, it is possible to find competitive equilibria in polynomial time for λ small enough by solving Problem 3, the conditions for solving Problem 3 in polynomial time being more general than those for finding a competitive equilibrium in polynomial time.

References

[1] Eytard J.-B., Akian M., Bouhtou M., Gaubert S., "A bilevel optimization model for load balancing in mobile networks through price incentives ", WiOpt 2017, 2017.

[2] Murota K., *Discrete convex analysis*, SIAM, 2003.

[3] Danilov V., Koshevoy G., Murota K. , "Discrete convexity and equilibria in economies with indivisible goods and money", Mathematical Social Sciences, 41:251-273., 2001.

Contributed session

Generalized Benders' Algorithm for Mixed Integer Bilevel Linear Optimization

Ted K. Ralphs*
Department of Industrial and Systems Engineering, Lehigh University, ted@lehigh.edu
Suresh Bolusani
Department of Industrial and Systems Engineering, Lehigh University, bsuresh@lehigh.edu
Sahar Tahernejad
Department of Industrial and Systems Engineering, Lehigh University, sat214@lehigh.edu

Abstract

We propose two related algorithms that take a generalized Benders' approach to solving general mixed integer bilevel linear optimization problems in which continuous and integer variables are present in both first- and second-level problems. The first of these algorithms is a pure Benders' algorithm, while the second integrates this approach into branch-and-bound to yield a so-called branch-and-Benders-cut algorithm.

Our pure Benders' algorithm generalizes the previously proposed algorithm for stochastic optimization problems with mixed integer recourse [1]. As with that algorithm, we make minimal assumptions, requiring only that first-level variables participating in the second-level problem be integer variables. As usual, the strategy is to project out the second-level variables in order to obtain a reformulation involving only first-level variables. This reformulation involves the so-called risk function (defined below), which is then iteratively approximated. The result is that the master problem is a single-level optimization problem involving only first-level variables, whereas the subproblem is the optimization problem resulting from fixing the first-level variables. We solve this latter problem by reformulating it as a mixed integer linear optimization problem.

More formally, we consider the following bilevel optimization problem.

$$\min \left\{ c^1 x + \Xi(x) \mid x \in X \right\}, \tag{MIBLP}$$

where Ξ is a *risk function* that encodes the part of the objective function that depends on the response to x in the second level, defined by

$$\Xi(x) = \min \left\{ d^1 y \mid y \in P_1(x), y \in \arg\min\{d^2 y \mid y \in P_2(x) \cap Y\} \right\} \forall x \in \mathbb{R}^{n_1}$$

where

$$P_1(x) = \left\{ y \in \mathbb{R}^{n_2}_+ \mid G^1 y \geq b^1 - A^1 x \right\} \textit{ and}$$
$$P_2(x) = \left\{ y \in \mathbb{R}^{n_2}_+ \mid G^2 y \geq b^2 - A^2 x \right\}$$

define parametric families of polyhedra containing points satisfying the linear constraints of the first- and second-level problems, respectively, with respect to a given $x \in \mathbb{R}^{n_1}$. X and Y are sets representing integrality requirements on the variables x and y respectively.

The master problem and subproblem are defined as follows.

$$
\begin{array}{llll}
(\mathbf{MP}): & \min c^1x + z & (\mathbf{\Xi(\bar{x})}): & \min d^1y \\
& \text{s.t. } z \geq \Xi(x) & & \text{s.t. } y \in P_1(\bar{x}) \\
& \quad x \in X & & \quad y \in \{\arg\min\ d^2y \\
& & & \qquad \text{s.t. } y \in P_2(\bar{x}) \cap Y\}
\end{array}
$$

In the master problem (**MP**), z is a variable representing the value of the risk function approximation. The subproblem is to evaluate $\mathbf{\Xi(\bar{x})}$ for a fixed $\bar{x} \in X$. This subproblem can be solved as a mixed integer optimization problem upon replacing the second-level optimality condition with a linear constraint $d^2y = \phi(b^2 - A^2\bar{x})$, where ϕ represents the second-level value function (and hence $\phi(b^2 - A^2\bar{x})$ is the optimal value of the second-level problem for fixed first-level solution $\bar{x}$). The constraint $z \geq \Xi(x)$ is then a Benders' "cut" that is iteratively strengthened in every iteration of the algorithm until convergence is achieved. Note that although this is conceptually a single constraint, it is linearized in practice, as we describe next, so that the master problem is a mixed integer linear optimization problem.

In (**MP**), $\Xi(x)$ is a lower-approximation for the risk function and can be constructed by extracting dual information from the nodes of the branch-and-bound tree resulting from solution of the subproblem using a technique similar to that proposed in [1] and [2]. Note that an upper-approximation $\bar{\phi}$ of ϕ (the second-level value function) is also required for constructing $\Xi(x)$. This is obtained by considering a certain restriction of the second-level problem. The approximation of the risk function we employ is then

$$\Xi(x) = \min_{t \in T}\{(b^1 - A^1x)\eta_t^1 + (b^2 - A^2x)\eta_t^2 + (\bar{\phi}(b^2 - A^2x))\eta_t^3 + \eta_t^4\},$$

where T represents the set of all leaf nodes of the branch-and-bound tree resulting from solution of the subproblem and $(\eta_t^1, \eta_t^2, \eta_t^3, \eta_t^4)$ is the dual solution at leaf node t. Since the function $\Xi(x)$ is nonlinear, the resulting Benders' cut would be nonlinear. We further linearize it by introducing binary variables in order to ensure that the master problem is a mixed integer linear optimization problem. This approximation function is guaranteed to yield a convergent algorithm.

Our branch-and-Benders-cut algorithm embeds the described Benders' algorithm in a branch-and-bound framework for mixed integer bilevel linear optimization problems. As with any branch-and-bound framework, the main components of the algorithm are bounding procedures, pruning rules, and branching strategy. These components are designed in accordance with the special features of these bilevel optimization problems, as described in [3]. Computational results using an open-source implementation will be presented.

Keywords: *Benders' algorithm, mixed integer bilevel linear optimization, value function, risk function*

References

[1] Ted K. Ralphs, and Anahita Hassanzadeh, "A Generalization of Benders' Algorithm for Two-Stage Stochastic Optimization Problems With Mixed Integer Recourse," COR@L Technical Report **14T-005**, 2014.

[2] Menal Güzelsoy, and Ted K. Ralphs, *Duality for Mixed-Integer Linear Programs*, International Journal of Operations Research **4** (2007), 118–137.

[3] Sahar Tahernejad, Ted K. Ralphs, and Scott T. DeNegre, “A Branch-and-Cut Algorithm for Mixed Integer Bilevel Linear Optimization Problems and its Implementation,” COR@L Technical Report **16T-015-R3**, 2017.

A Complementarity-Based Method for Solving Problems with Equilibrium Constraints with an Application to Energy Markets

Sauleh Siddiqui*
Departments of Civil Engineering and Applied Mathematics & Statistics, Johns Hopkins University, siddiqui@jhu.edu

Abstract This paper provides a new method to obtain strongly stationary points for mathematical and equilibrium problems with equilibrium constraints (MPECs and EPECs). The MPECs and EPECs considered have a complementarity problem at the lower-level, with the complementarity constraints linear. We use a nonlinear optimization algorithm which finds stationary points of an optimization problem with a bilinear objective function and linear constraints. Numerical evidence shows this method works for moderately sized EPECs, and outperforms current algorithms. Along with numerical examples, we provide an application to the US Biofuel market to show the usefulness of the algorithm.

MPECs and EPECs arise in engineering and economics applications including multi-leader-follower games, hierarchical optimization problems, networks, and robust optimization. However, solving large scale EPECs is computationally challenging due to the non-linearities in the problem structure and the disjoint, non-convex feasible regions. Current gradient-based methods rely on diagonalization or introduction of binary variables to obtain stationary points, which might or might not be local equilibria. We introduce a computational approach that does not use any of these techniques, but rewrites the equilibrium conditions as a complementarity problem using duality theory.

Mathematically, an EPEC is an equilibrium problem constrained by a lower-level problem that can be an optimization problem, a variational inequality problem, or, as this paper's focus, a mixed complementarity problem (MCP). Formally, having a function $\psi : \mathbb{R}^{n_z} \to \mathbb{R}^{n_z}$ an MCP is to find a vector $z \in \mathbb{R}^{n_z}$ such that:

$$z \geq 0; \psi(z) \geq 0; z^T \psi(z) = 0 \tag{1}$$

An EPEC is an equilibrium problem of n MPECs. An MPEC, for player i ($MPEC_i$) is given by

$$\begin{aligned} &\min_{x_i} f^i(x_i, \bar{x}_i) \\ \text{s.t.} \quad & g^i(x_i) \leq 0; h^i(x_i) = 0 \\ & 0 \leq G^i(x_i, \bar{x}_i) \perp H^i(x_i, \bar{x}_i) \geq 0 \end{aligned} \tag{2}$$

where the continuous variables $x_i \in \mathbb{R}^{n_{x_i}}$, are, respectively, the vector of $MPEC_i$'s decision variables, $\bar{x}_i \in \mathbb{R}^{n_{\bar{x}_i}}$ are a vector of decision variables for the other MPECs, $\{f^i(x_i, \bar{x}_i)\}_{i=1}^n$ are objective functions for each MPEC, $0 \leq G^i(x_i, \bar{x}_i) \perp H^i(x_i, \bar{x}_i) \geq 0$ are the complementarity conditions in the form of (1), and f, g, G, H are all continuously differentiable.

A solution to an EPEC is given by an equilibrium $x^* = (x_1^*, x_2^*, x_3^*, ..., x_n^*)$ among n players such that for each player i, $f^i(x_i^*, \bar{x}_i^*) \leq f^i(x_i, \bar{x}_i^*)$ for all $x_i \in \{x_i | g^i(x_i) \leq 0; h^i(x_i) = 0, 0 \leq G^i(x_i, \bar{x}_i) \perp H^i(x_i, \bar{x}_i) \geq 0\}$.

The general way to solve equilibrium problems is to take the first-order conditions of the individual optimization problems and combine all of them to solve a system of equalities and inequalities. However, when each individual optimization problem is an MPEC, the Karush-Kuhn-Tucker conditions are neither necessary nor sufficient. MPECs fail to satisfy several of the computationally tractable constraint qualifications, simply because of the complementarity constraint, which is bilinear. But we can still define stationary points for MPECs [1] as follows. Let $x^* = (x_1^*, x_2^*, x_3^*, ..., x_n^*)$ be a feasible point for $MPEC_i$ (2). Define the following active and biactive index sets:

$$I_g(x^*) := \{k | g_k^i(x^*)\} = 0; \qquad I_{0+} := \{k | G_k^i(x^*) = 0, H_k^i(x^*) > 0\}$$
$$I_{+0} := \{k | G_k^i(x^*) > 0, H_k^i(x^*) = 0\}; \qquad I_{00} := \{k | G_k^i(x^*) = 0, H_k^i(x^*) = 0\}$$

There are many stationarity concepts for MPECs, but we will restrict ourselves to the main ones. The feasible point $x^* = (x_1^*, x_2^*, x_3^*, ..., x_n^*)$ for $MPEC_i$ is said to be

- Weakly Stationary if there are multipliers $\lambda^g \in \mathbb{R}^m, \lambda^h \in \mathbb{R}^p, \gamma, \nu \in \mathbb{R}^q$ such that $\nabla_{x_i} f^i(x^*) + \sum_{k=1}^m \lambda_k^g \nabla_{x_i} g_k^i(x^*) + \sum_{k=1}^p \lambda_k^h \nabla_{x_i} h_k^i(x^*) - \sum_{k=1}^q \gamma_k \nabla_{x_i} G_k^i(x^*)$ $- \sum_{k=1}^q \nu_k \nabla_{x_i} H_k^i(x^*) = 0; \quad 0 \leq \lambda^g \perp -g^i(x^*) \geq 0; \quad h^i(x^*) = 0;$ $\gamma_k = 0 \quad (k \in I_{+0}(x^*)); \quad \nu_k = 0 \quad (k \in I_{0+}(x^*))$
- C-Stationary if it is weakly stationary and $\gamma_k \nu_k \geq 0$ for all $k \in I_{00}(x^*)$
- M-Stationary if it is weakly stationary and $\gamma_k > 0, \nu_k > 0$ or $\gamma_k \nu_k = 0$ for all $k \in I_{00}(x^*)$
- Strongly Stationary if it is weakly stationary and $\gamma_k \geq 0, \nu_k \geq 0$ for all $k \in I_{00}(x^*)$

Stationary Points for EPECs are defined in terms of the MPEC stationary points above: A vector is a Strongly Stationary (Weakly Stationary, C-Stationary, M-Stationary) point of the EPEC if for each $i \in 1, ..., n$, x^* is a Strongly Stationary (Weakly Stationary, C-Stationary, M-Stationary) point for each MPEC.

Note that all these stationarity concepts only differ by what is happening in the biactive set $I_{00}(x^*)$. The difference occurs in the values of the multipliers associated with G and H when both constraints are active. These differences can be realized by just taking the first-order conditions for each MPEC, and then simplifying. We show how not simplifying completely can lead to simpler optimality conditions, and a new format for developing algorithms for problems with equilibrium constraints.

Our objective is to prove that the proposed complementarity-based formulation of the EPEC outputs stationary points that are equilibria of the corresponding EPEC. Our approach is to express the MPEC and EPEC above as a complementarity problem by decomposing the dual variables associated with the bilinear constraints. We then solve the problem using techniques for optimizing a bilinear objective function over a polytope. The proposed computation strategy is justified because it involves solving one complementarity problem as opposed to using binary variables or solving a sequence of optimization problems with equilibrium constraints as current methods do. We leverage previous work by using existing theory of stationary Points for MPECs and EPECs as well as building on existing algorithms for optimization over polytopes. Note that while previous researchers have formulated complementarity-based formulations of EPECs [2],they contain nonlinear terms in the constraints, which means that standard complementarity problem theory cannot be applied for analysis. Instead of non-square, nonconvex formulations for each individual MPEC, we propose convex, square systems that converge to strongly stationary points. These algorithms will be tested against available techniques, and shown to be more computationally efficient through numerical and theoretical evidence.

The first main insight stems from decomposing the dual variables associated with the constraints G and H, which will guide new theory for optimality of MPECs and drive convergence to a strongly stationary point. Decomposing the duals associated with these constraints is a natural way to try and converge to stationary points. Consider the following complementarity problem. We have added surrogate variables p, q, u, v and have decomposed the dual variables γ_k and ν_k into their positive and negative parts. The surrogate variables ensure that any point that satisfies the problem below is a strongly stationary point.

Formulation of approach: Find $x^* \in \mathbb{R}^{n_x}, \lambda^g \in \mathbb{R}^m, \lambda^h \in \mathbb{R}^p$ and $u, v, \lambda^u, \lambda^v, \lambda^G, \lambda^H \in \mathbb{R}^q$ such that the following conditions are satisfied:
$\nabla_{x_i} f^i(x^*) + \sum_{k=1}^m \lambda_k^g \nabla_{x_i} g_k^i(x^*) + \sum_{k=1}^p \lambda_k^h \nabla_{x_i} h_k^i(x^*) - \sum_{k=1}^q (\lambda_k^G - \lambda_k^u) \nabla_{x_i} G_k^i(x^*) - \sum_{k=1}^q (\lambda_k^H - \lambda_k^v) \nabla_{x_i} H_k^i(x^*) = 0; \quad 0 \leq \lambda^g \perp -g^i(x^*) \geq 0; \quad h^i(x^*) = 0; \quad 0 \leq \lambda^G \perp G^i(x^*) \geq 0;$
$0 \leq \lambda^H \perp H^i(x^*) \geq 0; \quad 0 \leq p \perp u - G^i(x^*) \geq 0; \quad 0 \leq q \perp v - H^i(x^*) \geq 0;$
$0 \leq u \perp p + v \geq 0; \quad 0 \leq v \perp q + u \geq 0;$
$0 \leq \lambda^v \perp K(H^i(x^*)) - \lambda^u \geq 0; \quad 0 \leq \lambda^u \perp K(G^i(x^*)) - \lambda^v \geq 0.$

Note that in our formulation $\gamma_k = \lambda_k^G - \lambda_k^u$ and $\nu_k = \lambda_k^H - \lambda_k^v$ as we have decomposed the duals associated with the complementarity constraints into positive and negative parts. Computationally, this makes sense as any algorithm we improve will need information from both the duals. We can easily prove that if a standard complementarity problem algorithm (e.g., PATH) is applied to our formulation and outputs a solution, it will yield a strongly stationary point. Note that we have to choose a large enough constant K in this formulation, which is similar to the constant used in disjunctive constraints. This can be problem specific. We can formulate the above conditions without this constant K by the addition of further surrogate variables, but then this will only yield an M-stationary point.

More importantly, this problem does not contain any bilinear terms, and in many applications, this system of equations is linear. Thus, we should be able to use existing nonlinear optimization algorithms over polytopes [3], which will be presented as part of our work. Specifically, we will look at a multiplayer setting in the US Biofuel market as an example where we can apply our efficient solution technique for problems with equilibrium constraints

Keywords: *Stationarity, Complementarity, Energy*

References

[1] Kanzow, C., and A. Schwartz, *A new regularization method for mathematical programs with complementarity constraints with strong convergence properties*, SIAM Journal on Optimization **23** 2 (2013), 770–798.

[2] Leyffer, S., and T. Munson, *Solving multi-leader common-follower games*, **25** 4 (2010), 601–623.

[3] Hager, W., and H. Zhang, *An active set algorithm for nonlinear optimization with polyhedral constraints*, Science China Mathematics **59** 8 (2016), 1525–1542.

Modeling the waste-to-energy supply chain in a circular economy: a bilevel approach

Giorgia Oggioni*
Department of Economics and Management, University of Brescia, Italy,
giorgia.oggioni@unibs.it

Elisabetta Allevi
Department of Economics and Management, University of Brescia, Italy,
elisabetta.allevi@unibs.it

Luigi Boffino
Department of Management, Economics and Quantitative Methods, University of Bergamo, Italy, l.boffino@studenti.unibg.it

Maria Elena De Giuli
Department of Economics and Management, University of Pavia, Italy,
elena.degiuli@unipv.it

Abstract On December 2015, the European Commission adopted a package to support the EU's transition to a circular economy (see [1]). Looking beyond the current linear economy approach based on "take, make and dispose" industrial models, the circular economy foster the principle of the "reusing-reducing-recycling" raw materials to guarantee energy saving and to reduce greenhouse gas emissions. In a circular economy the value of products and materials is maintained for as long as possible. This "closing the loop" of product lifecycles through greater recycling and re-use represents an important step towards a sustainable economy. Waste plays an important role in the circular economy project since it can be adopted to produce energy. The waste and, more in general, the biomass can be considered as appealing sources of renewable energy that can be used to meet the environmental targets set out by the European Commission. We recall that, in 2005, the European Union has launched the European Emissions Trading System (EU-ETS), which is the first and largest cap-and-trade system in the world. This scheme imposes a cap on greenhouse gas emissions generated by power, industrial, and transport sectors (Directives 2003/87/EC and 2009/29/EC). The EU-ETS is one of the main tools designed to achieve the so-called 20-20-20 European targets by 2020. These ambitious targets aim at reducing CO_2 by 20% compared with 1990, at obtaining a 20% share of renewable-based energy in final energy consumption, and at reducing primary energy consumption by 20% compared with projected levels by improving energy efficiency. The accomplishment of these benchmark represents the first step towards the achievement of a low-carbon economy, as stated in the Energy Roadmap 2050.

In this paper, we consider the point of view of a power producer which manages a waste-to-energy supply chain and has to decide the amount of electricity to sell on the day-ahead market, taking into account its technology portfolio. In particular, we assume that the power producer disposes of technologies for the waste-to-energy treatment, renewables and conventional power plants. For this analysis, we propose a bilevel model, where the upper-level problem describes the operation decisions that the power producer takes in order to minimize its costs, while the lower-level problem represents the clearing of the day-ahead electricity market where other generators can participate (see [2] and [3]). Finally, a carbon regulation

on electricity production and uncertainty on renewable energy generation are considered.

Keywords: *Waste-to-energy, circular economy, uncertainty, bilevel programming.*

References

[1] European Commission, *Communication from the Commission of the European Parliament, the Council, the European Economic and Social Committe and the Committee of the Regions: Closing the loop-An EU action plan for the Circular Economy* (2015). Available at `http://eur-lex.europa.eu/legal-content/EN/TXT/?uri=CELEX:52015DC0614`

[2] Gabriel, S.A., A.J. Conejo, J.D. Fuller, B.f. Hobbs and C. Ruiz, "Complementarity Modeling in Energy Markets". Springer, New York, 2012.

[3] Fortuny-Amat, J. and B. McCarl, *A representation and economic interpretation of a two-level programming problem*, The Journal of the Operational Research Society **32** (1981), 783-792.

Contract Design in Electricity Markets in Presence of Stochasticity and Risk Aversion: A Two Stages Approach

Rossana Riccardi*
Department of Economics and Management, University of Brescia,
rossana.riccardi@unibs.it

Carlos Ruiz Mora
Department of Statistics, Universidad Carlos III de Madrid, caruizm@est-econ.uc3m.es

Arega Getaneh
Department of Economics and Management, University of Brescia, a.abate004@unibs.it

Abstract In the last three decades, an increasing number of countries worldwide have liberalized their electricity power sectors. Electricity prices are now determined by an equilibrium of supply and demand, which introduces a substantial price risk with volatilities much higher than those of equity prices. A big share of the total electricity in liberalized power markets is traded over the counter through bilateral agreements: it has been completed for some years in Scandinavia and the United Kingdom, is well under way in the United States and being embraced in most continental Western Europe,[1] Germany and the Netherlands are quite deregulated, followed by Spain. Italy is establishing power trading in a competitive environment. This represents a multi-billion spot market that is developing very quickly. And the same pattern of evolution as in the financial markets is being observed, with the growth of a variety of derivative instruments such as forward and futures contracts, swaps, plain-vanilla and exotic options ((i.e. non-standard) options like spark spread options, swing options and swap options).

In liberalized electricity markets, business risks can be effectively managed through contracts. Generators, retail suppliers and consumers can agree on prices, volumes, times and other conditions that create the desired certainty within the framework of the contract. In fact, liquid and effective markets for financial contracts improve competition by enabling sophisticated risk management. This, in turn, eases market access for new and smaller market players and contributes to ensuring that market power is not exercised. Most markets provide a framework for a liquid market in the day-ahead and real-time segments through market rules and design.

As electricity markets mature throughout the world, futures markets become more and more liquid and relevant for electricity trading. Futures markets allow trading products (mainly forward contracts and options) spanning a large time horizon, e.g., one month, while spot markets are typically cleared on an hourly basis throughout the time periods spanned by the futures market products. Thus, futures and spot markets interact and such interaction results in multi-market equilibria. [2].

Ruiz et al. in [2] provide an electricity equilibrium model involving a futures market and a collection of successive spot markets within the time span of the considered future derivatives.

[1]European reform was pursued at two parallel levels. First, under EU Electricity Market Directives, member countries were required to take at least a minimum set of steps by certain key dates toward the liberalization of their national markets. Second, the European Commission promoted efforts to improve the interfaces between national markets by improving cross-border trading rules, and to expand cross-border transmission links.

They conclude that the analytical results obtained are applied to a market involving different competition levels: monopoly, cartel, conjectural variation, Cournot and perfect competition.

However, the aforementioned research did not include uncertainty (both production cost uncertainty and demand uncertainty) and different contracts in their model. Therefore, using [2] as a departure, in this work, we develop a single spot market and a single futures market equilibrium models under uncertainty. Moreover, we add contract design (particularly, contracts for differences and two part tariffs contracts) between oligopolistic electricity producers in the presence of stochasticity with a two stages approach.

We assume a single futures market that is cleared prior to the spot market. Thus, the quantities sold in the futures market are to be delivered at the time when the single spot market takes place. Therefore, the novelty in our work is that we develop a simplified single spot market and introduce the two contracts in the presence of stochasticity, that mainly focuses on cost uncertainty and demand uncertainty, knowing that price and quantity are decided in advance. Contracts for differences and two part tariffs are assessed as a coordination mechanism for risk hedging and investment, which is also inspired by [1].

Finally, since operators are usually risk averse, a coherent risk measure is introduced to deal with both risk neutral and risk averse operators. To that end, this work has thrived to come across with the objective of developing simple and realistic models for the volatile and recently deregulated electricity market that can be recast as a mixed linear complementarity problem. Equilibrium conditions at each spot market are described as a function of the futures market decision variables, which in turn allows describing the equilibrium in the futures market implicitly enforcing equilibrium in each spot market.

Keywords: *Spot and future electricity markets, stochastic programming, risk aversion*

References

[1] Oliveira, Fernando S and Ruiz, Carlos and Conejo, Antonio J, *Contract design and supply chain coordination in the electricity industry*, European Journal of Operational Research **227(3)** (2013), 527–537.

[2] Ruiz, Carlos and Kazempour, S Jalal and Conejo, Antonio J, *Equilibria in futures and spot electricity markets* Electric Power Systems Research **7**, (2012), 1–9.

Contributed session

Mechanism Design and Allocation Algorithms for Network Markets with Piece-wise Linear Costs and Externalities

Alejandro Jofre*
CMM and DIM , Universidad de Chile , ajofre@dim.uchile.cl

benjamin Heymann
CRITEO, France , benjheymann@gmail.com

Abstract Motivated by the problem of market power in electricity markets we introduced a mechanism design in a previous work for simplified markets of two agents with linear production cost functions. In standard procurement auctions, the market power resulting from the quadratic transmission losses allows the producers to bid above their true value, which are their production costs. The mechanism proposed in the previous paper, based on Stackelberg models, reduces the producers' margin to the society's benefice. In this paper, we extend those results to a more general market made of a finite number of agents with piecewise linear cost functions, which makes the problem more difficult, but simultaneously more realistic. We show that the methodology works for a large class of externalities. We also provide an algorithm to solve the principal allocation problem.

Keywords: *energy market, pricing, principal agent model, Stackelberg, market power, mechanism design, network economics*

Demand-Response-Oriented Electricity Price Setting for Residential Buildings

Juan A. Gomez-Herrera *
Department of Mathematical and Industrial Engineering, Polytechnique Montreal & GERAD, juan.gomez@polymtl.ca

Miguel F. Anjos
Department of Mathematical and Industrial Engineering, Polytechnique Montreal & GERAD, miguel-f.anjos@polymtl.ca

Luce Brotcorne
Team INOCS, Inria Lille-Nord Europe, luce.brotcorne@inria.fr

Abstract

The power systems operation requires a continuous and real-time balance between generation and consumption. In this context, satisfying peaks of demand can be a challenging and expensive task for systems operators and utilities, since following a fast increase in demand normally leads to the use of more flexible but more expensive generation technologies.

Demand response (DR) is increasing in popularity as an effective way to support the supply-demand balance. DR programs are designed to encourage end-users to modify their normal consumption profile in a way that contributes to the operation and the economical feasibility of the system or even to avoid critical situations such as blackouts.

The potential benefits of DR increase with the integration of renewable generation to power systems. In this scenario, the traditional generation accounts for the net demand, which is defined as the electricity demand minus the renewable generation. Since the renewable resources are typically intermittent, they can bring sudden and even more pronounced peaks in the net demand curve.

Different DR programs are offered to different consumers. Predefined consumption reduction agreements are traditionally oriented to large customers while pricing programs are common for residential users around the world. Pricing programs offer the end-users a set of energy tariffs that approximates the generation cost function. This way the consumers are encouraged to shift their load to periods when they can profit from lower prices.

A time of use (TOU) program introduces at least two different energy prices during the day, accounting traditionally for the on-peak and the off-peak hours. This idea has been widely included in optimization models for demand side management (DSM) for smart buildings. These DSM models normally minimize the user's cost function by planning or scheduling the use of appliances and other devices, considering user's preferences, user's dissatisfaction, etc. As expected, the optimization models succeed in shifting the consumption from on-peak to off-peak periods. This approach makes sense in a local perspective, for up to a few users, but raises other questions about how the shifting behaviour in a large population can affect the generation cost function. In other words, large shifted consumption might lead to rebounds peaks.

This question was partially addressed in [1] using a time and level of use (TLOU) policy. Similarly to TOU, TLOU presents a set of different energy prices but it includes a second set of energy prices that apply to consumption above a certain capacity limit. This approach brings a power-wise perspective by penalizing high consumption per time frame. The authors in [1] show that a proper definition of the tariffs and capacity limits for the TLOU can avoid rebounds peaks for small populations using optimization-based DSM systems.

This work looks into the setting of TLOU tariffs and capacity limits for large populations. It uses a bi-level optimization problem where the utility is the leader who minimizes the generation cost and the end-users are the followers minimizing their individual cost function by utilizing a DSM module. We present experimental results and a sensitivity analysis based on realistic data to validate the effectiveness of the proposed approach.

Keywords: *Bi-level programming, Demand response, Smart Grids*

References

[1] Gomez, J. A., and M. F. Anjos, *Collaborative demand-response planner for smart buildings*, Cahiers du Gerad (2017), G-2017-15.

Contributed session

Bi-level programming approaches to optimize dynamic electricity tariffs considering demand response

Carlos Henggeler Antunes*
Dept. of Electrical and Computer Engineering of University of Coimbra / INESC Coimbra, Portugal, ch@deec.uc.pt

Maria João Alves
CeBER and Faculty of Economics of University of Coimbra / INESC Coimbra, Portugal , mjalves@fe.uc.pt

Abstract The electricity sector has been open to retail competition, including to residential customers. Retailers procure electricity in wholesale markets or through bilateral contracts, and then offer flat or (slightly) variable time-of-use pricing schemes to their residential customers for the contract period (e.g., one year), thus managing the risk involved. Those price signals do not reflect varying generation costs, namely in systems with a significant penetration of intermittent renewable generation, and network conditions, such as congestion. Dynamic tariffs, i.e. energy prices varying possibly with significant magnitude in short periods of time, have the potential to modulate demand thus contributing to improve the overall system efficiency, lowering peak generation costs, facilitating the penetration of renewable sources, reducing network losses and deferring network reinforcement investments, while offering economic benefits to customers. These price incentives will motivate consumers to engage in different consumption patterns, namely by using the flexibility they generally have in the operation of some appliances (end-use loads) through adequate demand response actions, which may turn beneficial to the grid. After receiving tariff information some time in advance (e.g., one day), the consumer's home energy management system (HEMS) is able to respond by scheduling the operation of laundry machines, electric vehicle charging, etc. Different operation schedules represent distinct trade-offs between the minimization of the electricity bill and the minimization of the discomfort associated with postponing/anticipating load operation to different periods. HEMS have communication capabilities to receive grid information (prices and possibly emergency requests), are parameterized with the customer's energy usage preferences and are able to control loads through on/off plugs.

In this setting, electricity retailers should design commercial offers of dynamic tariffs to maximize profits, i.e. its optimal pricing scheme, subject to consumer's response. Consumers want to minimize costs, also taking into account comfort requirements and preferences associated with the operation of loads in appropriate time slots. There is a hierarchical relation between retailers and consumers: retailers determine prices and consumers react by scheduling their loads accordingly. Since HEMS, on behalf of consumers, are able to deviate consumption of shiftable loads (dishwashers, laundry machines, etc.) to lower price periods, the retailer's profits may decrease due to consumer's decisions. This interaction can be modeled as a bi-level (BL) optimization problem.

This work presents a BL optimization model to determine the optimal pricing scheme to be established by the retailer (upper level decision maker, the leader) to maximize profits, i.e., the revenue from selling energy to consumer minus the cost of purchasing the energy in the spot market, and the load schedule adopted by the consumer (lower level decision maker, the follower) under that price setting to minimize the electricity bill and minimize the discomfort associated with the operation of appliances outside the most preferred (or habitual) periods. Discomfort is assessed by means of penalty coefficients associated with time slots; the more or less stringent penalties characterize the degree of willingness of consumers to engage in demand response regarding each appliance. This is a semi-vectorial bi-level (SVBL) model, with a single objective in the upper level and two (conflicting) objective functions in the lower level. The lower level optimization problem is formulated as a multi-objective mixed-integer linear programming (MOMILP) model. This SVBL model is an extension of the single objective BL optimization model presented in [1].

Upper level decision variables are the price charged by the retailer to the consumer during each sub-period of the planning period (dynamic tariffs during one day). The lower level decision variables represent whether a given load j is "off" or "on" at time t of the planning period and stage r of its operation cycle; these variables are used to determine the power requested to the grid by each appliance in each stage (auxiliary continuous variables). The binary decision variables are defined only for t in the time slot allowed for the operation of each load according to the consumer's comfort requirements. Consumer's decisions are thus modeled at appliance level, considering typical operation cycles, as well as information about time slots preferred for operation of shiftable loads, which is essential to obtain realistic load characterization and demand response actions.

Upper levels constraints define the upper/lower bounds for the energy prices charged to the consumer in each sub-period and set an average price in the planning period, as a surrogate to model competition in the electricity retail market. Lower levels constraints impose that the contracted power is not exceeded at any time t of the planning period and consistency conditions regarding load operation within its allowed time slot.

The SVBL model is tackled using a hybrid approach consisting of a genetic algorithm (GA) for the upper level problem and an exact MILP solver to solve surrogate scalar problems (i.e. combining both objective functions) at the lower level for a given electricity price setting.

The consideration of multiple objective functions at the lower level introduces further challenges regarding computational difficulties to ensure that solutions are actually non-dominated and the need for the retailer to examine a set of extreme outcomes resulting from his optimistic/pessimistic stance and the possible consumer's reaction. In this setting, the computation of four extreme (optimistic, pessimistic, deceiving and rewarding) solutions offers comprehensive information for the retailer's decision process. The optimistic solution indicates the leader the maximum profit when the follower's decision for each upper level variables setting is the best for the leader. The deceiving solution results whenever the leader makes an optimistic decision and the follower's reaction is against the interests of the leader, i.e. it results from a failed optimistic approach. The pessimistic solution gives the maximum profit for the leader when the follower's decision for each upper level variables setting is the worst for the leader. The best return of a pessimistic approach is the rewarding solution, which is obtained whenever the leader takes a pessimistic approach and the follower's reaction is the most favorable to the leader. These four extreme solutions will be exploited since they offer the retailer useful insights about the ranges of possible profit values under different perspectives of analysis. This is particularly interesting if the leader has no information about the trade-offs the consumer is willing to make between the economic and comfort dimensions after knowing the electricity prices set by the retailer.

The presentation will address the following main topics:

- single objective BL mathematical model and its extension to the SVBL model to study the interaction between a retailer designing dynamic tariffs and responsive consumers who weigh the electricity bill against comfort requirements;
- a hybrid algorithmic approach encompassing a GA and a MILP solver;
- the exploitation of the optimistic, deceiving, pessimistic and rewarding solutions for decision support purposes;
- the incorporation of the consumer's preferences to compute other non-dominated solutions trading-off the economic and comfort dimensions.

Keywords: *Semivectorial bilevel optimization, Multi-objective optimization, Genetic algorithms, Hybrid approaches, Demand response, Electricity retail markets*

References

[1] Alves, M.J., Antunes, C. H., Carrasqueira, P., *A hybrid genetic algorithm for the interaction of electricity retailers with demand response*, G. Squillero and P. Burelli (Eds.): EvoApplications 2016, Part I, LNCS 9597, 459–474, Springer, 2016.

Heuristic Algorithms for a Bilevel Service Network Design and Pricing Model

Christine Tawfik*
QuantOM, HEC Management School, University of Liege (ULg),
christine.tawfik@uliege.be

Sabine Limbourg
QuantOM, HEC Management School, University of Liege (ULg),
sabine.limbourg@uliege.be

Abstract Pricing decisions play a crucial role in positioning a certain product or service. They involve a non-trivial tradeoff between increasing revenues and keeping the service economically appealing, which requires a wise management of the underlying costs and a knowledge of the market situation in terms of the customers' alternative options. In the context of freight transport services, this stream of problems is equally relevant in the rise of reduced policy interventions (e.g., deregulation, privatisation, etc.), and particularly for rail-based transport, where operators are typically required to make months-long slots' reservation requests from the infrastructure managers in light of their expected demands. Other related system examples could include trucking and container shipping lines.

This work is devoted to jointly examining the intertwined tactical problems of designing freight carrying services and determining their associated prices as observed by the shipper firms. We put forward a bilevel model, where the main decision maker (a leader) is portrayed at the upper level, as a freight transport operator seeking to maximize his/her profits by setting the services' tariffs and selecting their subsequent operating frequencies. At the lower level, the shippers (followers), faced with itineraries composed of the leader's services and an always present competition's alternative, react to the leader's decisions in a costs' minimization fashion. As a more precise application context, we consider the leader and the competition as providers of freight-carrying intermodal and trucking services, respectively. Intermodal is used with the interpretation of a multimodal chain of transport services, where the long haul is performed by environment-friendly transport modes with higher payloads, such as rail or inland waterways (IWWs), and road is used for the shortest possible initial and final parts of the journey. The market is thus assumed to consist of small customer shippers seeking to take advantage of freight consolidation, by choosing to send their demands between the proposed intermodal itineraries and the trucking alternative, with the possibility of *splitting* their volumes over several paths.

A linearised mixed-integer programming (MIP) formulation is developed, extending both the path-based multicommodity service network design model in [1] and the bilevel joint design and pricing problem in [2]. Capacitated services and discrete design variables are considered depicting their frequency decisions, as opposed to the classical open-close approach, adding in turn a considerable complexity to the problem. The linearisation steps yield a formulation involving big M constants, which is further enhanced by studying feasible theoretical

bounds. The model is invoked on real-world instances representing large-scale demands at the European continental level, using the classic solver CPLEX. The obtained results suggest a room for improvement, especially in terms of the computation time.

Therefore, we discuss two heuristic approaches to solve the proposed model, addressing its main point of difficulty stemming from the network design part. The first algorithm is based on the idea of starting with an initial, yet costly, services' schedule that is able to accommodate all shipping demands, then, in an iterative manner, decrease the frequencies of those services that do not considerably or positively contribute to the leader's generated gain. The initial solution is obtained from a version of the model with the integral design variables being relaxed: a problem that can converge to optimality in a short amount of time. Afterwards, an iterative procedure is invoked to solve a set of restricted MIP formulations of the model, where a set of the services' frequencies is fixed to the values obtained from the previous iteration. At each step, the choice criteria to decrease the frequency of a certain service are based on its load factor or profit margin's level. Checking techniques are additionally implemented to prevent the algorithm from re-visiting older services' schedules. Finally, the procedure is stopped when objective stagnation is reached and the solution associated with the maximum obtained objective value so far is returned.

Second, a heuristic algorithm is designed based on the Lagrangean relaxation framework. More specifically, we identify the service capacity constraints as the *complicating* ones and append them to the objective to obtain a Lagrangean function. The subgradient optimization algorithm is then applied to find the values of the Lagrange multipliers, where a lower bound on the solution is obtained by an initial constructive heuristic. Due to the particular costs' structure in the objective function, the mutlipliers are initialized with their theoretical lower bounds, instead of an arbitrary value. As in classical Lagrangean heuristics, we seek to restore the feasibility of the obtained solution through a procedure that re-routes the excess volumes on the services whose capacity is violated.

The two developed algorithms are tested on real-world instances with varying sizes and logistics properties. The main points of differences are highlighted, as well as the potential enhancements to achieve solutions with higher qualities.

The underlying research in this work is conducted in accordance with the project BRAIN-TRAINS, concerned with the future of rail freight intermodality in Belgium and Europe, and funded by the Federal Science Policy according to the contract n. BR/132/A4/BRAIN-TRAINS.

Keywords: *network design, network pricing, lagrangean relaxation*

References

[1] Crainic, T. G., *Service network design in freight transportation*, European Journal of Operational Research **122** (2000), 272–288.

[2] Brotcorne, L., Labbe, M., Marcotte, P., and Savard, G., *Joint design and pricing on a network*, Operations Research **56** (2008), 1104–1115.

Exact Algorithms for Fixed Charge Network Design Problem with User-Optimal Flows

Ikram Bouras[*]
Lirmm, University of Montpellier, ikram.bouras@lirmm.fr

Rosa Figueiredo
LIA, University of Avignon, rosa.figueiredo@univ-avignon.fr

Michael Poss
Lirmm, University of Montpellier, michael.poss@lirmm.fr

Fen Zhoo
LIA, University of Avignon, fen.zhou@univ-avignon.fr

Abstract We study the fixed charge network design problem with user-optimal flows, which is modeled as a bi-level programming problem. We propose a new Binary Integer Programming (BIP) formulation. This formulation is used to solve the problem by cutting plane and branch-and-cut algorithms. Numerical experiments are performed on realistic instances as well as on random data sets generated with different criteria to examine the difficulty of the instances. The results also show the efficiency of our proposed approaches.

Keywords: *Network design, Bi-level programming, Cutting plane, Branch-and-Cut.*

1 Introduction

The Fixed Charge Network Design Problem (FCNDP) consists in selecting a subset of edges from a given network, in such a way that a set of commodities can be transported from its origins to its destinations. The objective is to minimize the sum of fixed costs (depending on the selected edges) and variable costs (depending on the flow of commodities on the edges). There are several variants of FCNDP in the literature, each of which considers a particular objective function or takes into account specific additional constraints.

Herein, we are interested in a specific variant of the FCNDP, namely Fixed Charge Network Design Problem with User-Optimal Flows (FCNDP-UOF) with applications in the transportation network design problem for transportation of hazardous materials. This problem combines the FCNDP with multiple shortest path problems.

2 Problem description

We consider a transport network modeled by an undirected graph $G(V, E)$, where V represents the set of facilities and E represents the connections between them, which are uncapacitated.

There is a set of K commodities to be delivered in the transport network $G(V, E)$, where each edge $e = \{i, j\}$ is associated with several parameters: a length c_{ij}, a fixed cost f_e and a variable cost g_{ij}^k for each commodity k. Our objective is to design a subnetwork with minimum total cost to transport K commodities such that each commodity $k \in K$ with a flow ϕ^k can be delivered through the shortest path from its origin $o(k)$ to its destination $d(k)$ in this subnetwork. The FCNDP-UOF has been first formulated as a bi-level integer programming problem [2], and then it is further reformulated as one-level mixed integer programming models through replacing the second level by its optimality conditions in [2] and [3]. More recently, a cutting plane algorithm based on a set of valid inequalities is proposed in [1].

3 Proposed methods

We propose in this work a new Binary Integer Programming model (BIP) for the FCNDP-UOF. To this end, we replace the objective function of the second level by valid inequalities eliminating infeasible solutions. Denoting the set of all paths in the graph by $\mathcal{P}$, our BIP formulation is given as follows:

$$\min \quad \sum_{e\in E} f_e y_e + \sum_{k\in K} \sum_{(i,j)\in E} \phi^k g_{ij}^k x_{ij}^k \tag{1a}$$

$$\text{s.t.} \quad \sum_{(i,j)\in\delta^+(i)} x_{ij}^k - \sum_{(i,j)\in\delta^-(i)} x_{ji}^k = b_i^k, \qquad \forall i \in V, \forall k \in K, \tag{1b}$$

$$x_{ij}^k + x_{ji}^k \le y_e, \qquad \forall e \in E, \forall k \in K, \tag{1c}$$

$$\sum_{(i,j)\in E} x_{ij}^k \le c(P') + (|P'| - \sum_{e\in P'} y_e)M, \qquad \forall P' \in \mathcal{P}, k \in K, \tag{1d}$$

$$x_{ij}^k \in \{0, 1\}, \qquad \forall i, j \in E, \forall k \in K, \tag{1e}$$

$$y_e \in \{0, 1\}, \qquad \forall e \in E, \forall k \in K. \tag{1f}$$

Where, y_e is a binary variable takes value 1 if the edge e is chosen as a part of the solution network, and the variable x_{ij}^k represents the flow of commodity k through the arc (i, j).

In (1b), we have flow conservation constraint. Constraint (1c) does not allow flow to enter arcs whose corresponding edges are closed, (1d) eliminates all paths with a cost bigger than the cost of an opened path in the network. Finally, (1e) and (1f) impose the variables x_{ij}^k and y_e to be binary.

Based on this formulation, we implemented two methods to solve the problem. The first one is a cutting plane algorithm in which we solve the initial problem defined in (2) below so that we can obtain a subnetwork y^*, and a set of paths x^* for all origin-destination pairs.

$$\min \left\{ \sum_{e\in E} f_e y_e + \sum_{k\in K} \sum_{(i,j)\in E} \phi^k g_{ij}^k x_{ij}^k, \quad s.t. \quad (1b), (1c), (1e), (1f) \right\} \tag{2}$$

Then, they check if x^* is the shortest path in y^*. If it is the case, then (x^*, y^*) is optimal for the FCNDP-UOF. Otherwise, a valid inequality (1d) is added to problem (2) for the commodities with infeasible paths. We repeat the procedure until an optimal solution is obtained.

The above approach can be expensive in terms of number of iterations to get the optimum. Thus, our second strategy is to combine the two methods together to form a "branch-and-cut" algorithm, known to be an efficient way to handle such constraints. The latter algorithm solves the problem (2) by a branch-and-cut procedure that adds the inequalities to each integral node violating the shortest path constraint.

Our second contribution lies in assessing the difficulty of the instances used in the simulations. To this end, we categorize the instances by calculating the angle α between the objective function vectors of the first and the second levels.

We have coded and compared the performance of six algorithms listed as follows:

B&B1, B&B2: One-level formulations from [2] and [3], respectively;

CP1, B&C1: The cutting plane and branch-and-cut algorithms proposed in this work;

CP2, B&C2: The Cutting plane algorithm proposed by [1] and branch-and-cut algorithm coded using the same valid inequalities.

Numerical tests are performed using both random and real data sets. Tab. 1 compares the number of instances solved to optimality in 3600s by the six methods. The table shows that the two proposed cutting plane approaches have a comparable performance and they solves more instances optimally than the others. Our cutting plane algorithm **CP1** has also the best CPU times for the both data sets.

Table 1: Number of instances solved to optimality

	Total	B&B1	B&B2	CP1	CP2	B&C1	B&C2
$0° \leq \alpha \leq 10°$	135	100	109	110	110	110	109
$40° \leq \alpha \leq 50°$	135	88	89	95	95	94	92
$80° \leq \alpha \leq 90°$	135	88	97	105	105	102	102
Ravenna data	7	6	3	7	7	4	4
Sum	412	283	301	317	317	310	307

4 Conclusions and future work

For the FCNDP-UOF, we proposed a new BIP formulation, which is solved by both a cutting plane and a branch-and-cut algorithms. Numerical results demonstrate that the cutting plane algorithm based on the proposed BIP formulation outperforms the other algorithms. The instances with an angle of $40° \leq \alpha \leq 50°$ are the most difficult. For future work, we intend to work on exact approaches for the case of the FCNDP-UOF, where the capacity constraint on edges is considered.

References

[1] Gzara, Fatma, *A cutting plane approach for bilevel hazardous material transport network design*, Operations Research Letters, 41(1), 40-46.

[2] Kara, Bahar Y and Verter, Vedat, *Designing a road network for hazardous materials transportation*, Transportation Science, 2004, vol. 38, no 2, p. 188-196.

[3] Mauttone, A and Labbé, M and Figueiredo, R, *A Tabu Search approach to solve a network design problem with user-optimal flows*, VI ALIO/EURO Workshop on Applied Combinatorial Optimization, 2008.

Contributed session

Fair Risk Distribution for the Multi-mode Hazmat Transport Network Design Problem

Pirmin Fontaine*
TUM School of Management, Technical University of Munich, pirmin.fontaine@tum.de

Teodor Gabriel Crainic
CIRRELT and Department of Management and Technology, Université du Québec à Montréal, teodorgabriel.crainic@cirrelt.net

Michel Gendreau
CIRRELT and Département de Mathématiques et Génie Industriel, École Polytechnique, Montréal, michel.gendreau@cirrelt.net

Stefan Minner
TUM School of Management, Technical University of Munich, stefan.minner@tum.de

Abstract

1 Introduction

Hazardous material accidents can have tremendous consequences for the population. One of the worst accidents of this kind in recent history happened in July 2013 in Lac-Mégantic, QC in Canada. A driver-less train with 72 tank cars of petroleum crude oil derailed in the city center and caused the death of at 47 persons. Moreover, at least 30 buildings and 115 businesses were destroyed and it took almost 2 days to control the fire. But the transport of hazardous materials is essential not only for industrial countries like Canada, Germany and United States but also for developing countries. The four frequently shipped hazardous materials are - with 80% of the transported volume in Canada - crude petroleum, gasoline, fuel oils and non-metallic minerals. In 2012, 2,580 million tons of hazardous materials where shipped throughout the United States. 59.4% where transported by truck, 4.3% by rail, 11% by water and 24.3% by pipeline in single mode transportation. Only 1% was transported by more than one mode. In Canada, the rail has a much higher relevance. In 2012, 26.1 million tons were transported by rail and 107.4 million tons by truck. The different structure of the network in Germany, which is, compared to North America, very dense, is also reflected in the share of used transportation modes: In 2010, 56 million tons were transported by maritime transport, 48 million tons on inland waterways, 63 million tons by rail and 140 million tons by trucks. Therefore, the consideration of different transportation modes is essential for the risk calculation whenever regulating the transport of hazardous materials. Yet, the different streams of research investigate the transportation of hazardous material either on roads [1] or on rail [2] separately.

We aim to fill this gap by considering different transportation modes within the Hazmat Transport Network Design Problem (HTNDP). Moreover, classical risk definitions neglect that the risk in a population center is influenced by all links in the area of the population center. This is certainly true when the network consists of different modes, but also when, for example, several roads enter or pass by a city. Therefore, we further introduce a new population-based risk definition to evaluate the risk in population centers. For the fair distribution of risk among the population, different objective functions are introduced and compared. The proposed multi-mode multi-commodity bilevel formulation is transformed into a mixed-integer linear program and evaluated in a numerical study to show the benefits of the concept compared to classical risk definitions.

2 Problem definition

The society requests a fair distribution of risk over the population and the government or authority wants to achieve that by deciding if a link of the network is allowed for the transportation of hazardous materials or not. Therefore, the authority has to anticipate the reaction of the carriers who are shipping commodities with origin, destination, transportation amount and hazardous material type through the network. The carriers minimize their transportation costs subject to demand satisfaction by deciding on the transportation path of the commodities. Compared to previous studies, we not only consider different transportation modes in the network, but allow also the carriers to decide on the transportation mode of the shipment.

2.1 Population-based risk definition

In classical definitions of the Hazmat Transport Network Design Problem (HTNDP), the risk is associated with the links of the network [3]. The risk is defined as the accident probability multiplied by the transported amount of goods and the population in the area of the link. However, this formulation only distributes the risk fairly as long as only one arc is in the risk area of a population center. Otherwise, even if the risk is equally distributed on the links, some population centers can experience a much higher risk than others.

Therefore, we introduce a new definition of risk, which is based on population centers and not on arcs. A population center can be either a whole town or city or, for larger urban areas, a part of the city. We define the risk of a population center as the sum over all shipments on all modes in the influence range of the center weighted with the accident probability of the link and an influence factor. The influence factor depends on the transported hazardous material and the distance between the link and the center. It is higher when an arc is closer to the population center or the transported material is more dangerous.

To equilibrate the risk over these population centers, we use the new risk definition and investigate different objective functions for the HTNDP. Besides minimizing the overall risk in the network and the maximum risk in a population center, we consider non-linear risk functions like the maximum and average risk difference of population centers and the maximum and average deviation to the mean to achieve a fairer distribution of risk.

3 Model, solution method and numerical study

The problem is modeled as a bilevel problem [2] on a graph, where the nodes are important points in the network and the links are transportation modes like roads or rail tracks between these nodes. The government is the leader and the carriers represent the follower. The follower problem is a capacitated multi-mode transportation problem while the leader problem is the minimization of one of the various risk functions.

Due to the special structure of the bilevel problem with the binary leader variables (to allow the usage of an arc or not) and continuous follower variables (flow variables), the follower problem is replaced by its Karush-Kuhn-Tucker conditions and transformed into a mixed-integer program. The non-linear objective functions of Section 2.1 are reformulated into equivalent linear formulations by introducing auxiliary variables and auxiliary constraints to solve the problem by Xpress.

In the numerical study, we compare our model with the classical model from the literature and proof the concept of the multi-mode HTNDP and the population-based risk definition. Further, we analyze the trade-off between risk minimization and risk equilibration using a biobjective function and show that both objective functions have a convex correlation. Therefore, a significant improvement in risk distribution can be achieved at the cost of just a small increase in total risk. The cities with high risk benefit from the risk redistribution in the beginning. However, strong equilibrations just penalize cities with low risk. Moreover, compared to classical approaches in the literature, we achieve a better risk distribution among the population without increasing the total risk.

4 Conclusion

The contributions the extension of the HTNDP to a multimode transportation problem. Moreover, a new population-based definition of risk and equity risk measures for hazardous material shipments to fairly distribute risk in a network. The model is used to compare the different risk equilibration measures and to give insights on the trade-off between risk equilibration and risk minimization Finally, a comparison to existing models from the literature (single-mode and maximum arc risk equilibration) shows the benefits of our approach.

Keywords: *hazardous materials transportation; risk equilibration; multimodal, multicommodity network design*

References

[1] Kara, B. Y., and V. Verter, *Designing a road network for hazardous materials transportation*, Transportation Science, **38(2)** (2004), 188–196

[2] Verma, M., V. Verter, and M. Gendreau, *A tactical planning model for railroad transportation of dangerous goods*, Transportation Science, **45(2)** (2011), 163–174

[3] Alp, E., *Risk-based transportation planning practice: Overall methodology and a case example*, Infor, **33(1)** (1995), 4

Towards synchromodal planning: a joint capacity booking and adaptive routing problem

Alberto Giudici*
Department of Technology and Operations Research, Eramsus University Rotterdam,
`giudici@rsm.nl`

Tao Lu
Department of Technology and Operations Research, Eramsus University Rotterdam,
`lu@rsm.nl`

Clemens Thielen
Department of Mathematics, University of Kaiserslautern, `thielen@mathematik.uni-kl.de`

Rob Zuidwijk
Department of Technology and Operations Research, Eramsus University Rotterdam,
`rzuidwijk@rsm.nl`

Abstract Container transport planners in the hinterland of a port have to deal with uncertainties in transit time and information availability while managing the a-priori planning and real-time routing of reliable and cost efficient flows of containers. In this work, we model the problem faced by a central network orchestrator that aims at planning synchromodal transport for batches of containers on a short-term planning horizon. In the network, different transport modes (barge, train and truck) are available and characterized by different unit costs, capacity and stochastic transit time distribution. The model we propose is based on a planning problem faced by freight forwarders and contributes to the understanding of synchromodal planning by introducing bilevel problem for the joint capacity booking and adaptive routing of container on a transport network.

From this perspective, we capture the characteristics of planning synchromodal transport into an high-level model and gain insights into the value of such a planning approach.

Synchromodal transport is a novel concept suggesting an organization of inland transport based on two conditions: *a-modal booking* and *adaptive mode, and route, choice* (cf. [1]). The former requirement prescribes that transport mode is not pre-determined at the moment transport is purchased but kept free for further definition, while the latter implies that routing and transport mode decisions can be modified in real-time based on updated information on the status on the network. Those two aspects are the most relevant for our work, other characteristics of synchromodality can be found reviewed in [6].

Planning models for synchromodal transport have already been proposed but either consider problems on medium term planning horizons or the specific characteristics of certain companies. An integrated service schedule design approach is described in [1], while an online optimization problem for assignment of containers to transport services is discussed in [7] for the case of Rotterdam based terminal and intermodal operator ECT.

The model we propose is grounded in the stream of research on the adaptive shortest path problem. In [3], least expected transit time are shown to require a strategy to be found, rather than a static path. In [4], uncertainty in arc transit times is considered dependent on the time the arc is traversed and an algorithm to find the adaptive least expected path is proposed. Finally, in [5], a Markov Decision Process (MDP) models the adaptive routing problem for a single passenger in a bus transport network with stochastic transit time where information on buses' position is explicitly taken into account.

We propose a bilevel programming model were a-priori transport planning and adaptive routing are considered simultaneously as decisions taken prior to the execution of a transport plan create the freedom required to react optimally to uncertainty in transit times. We model the adaptive decision making by defining a MDP for an (s,t)-flow problem on a network where arcs are characterized by stochastic, time-varying transit time and upper capacities. Knowing the position of each container in the network, aim of the MDP is to determine an adaptive routing strategy using available capacity that maximizes the expected number of containers arriving at the sink node t earlier than a given deadline T. This MDP constitutes the lower level while selection of available capacities for the adaptive routing occurs at the upper level. At this level, the aim is that of minimizing the total capacity cost, as capacity booking generates costs. Bilevel optimization captures "the hierarchical relationship between two autonomous, and possibly conflictual, decision makers"([2]). This opposing relation can be found in our approach as the less capacity is selected at the upper level, the lower will be the objective value of the MDP. The MDP determines the optimal strategy to follow in real-time to route containers to their destination while the upper level minimizes the requirements in capacity to be allocated to the lower level(i.e. the MDP).

Our model contributes, therefore, to the current literature by modelling an adaptive problem on a stochastic, time-varying network when multiple units of flow are considered, rather than a single one as in the adaptive path problems. Moreover, this problem takes into account the a-priori allocation of capacity on different transport services to create room for adaptively react to uncertainty.

Let $G = (V, R)$ be a directed graph, V is the set of nodes and R the set of arcs representing transport services. Let $s, t \in V$ be the source node and the sink node, respectively, and let $K \in \mathbb{N}$ be the number of containers[1] to be routed to t earlier than a given deadline $T \in \mathbb{N}$. Let $\mathcal{T} = \{0, 1, \dots, T\}$ be the discretized, and possibly relabelled, time horizon. Let $c_r \in \mathbb{N}$ be the per-unit of capacity cost for allocating capacity to the adaptive planning phase on arc $r \in R$. We define time-dependent upper capacities $u_{r,\theta} \in \mathbb{N}_{\geq 0}$ to represent departure times ($r \in R$ and $\theta \in \mathcal{T}$). For arc $r \in R$ and departure time $\theta \in \mathcal{T}$ such that $u_{r,\theta} > 0$, we let $\tau_{r,\theta}$ be $\mathcal{T}$-valued random variable representing the stochastic arrival time for service r departing at time θ. Let $\kappa \geq 0$ be minimum level of reliability required.

Aim of this problem is to select capacities $\vec{x} = (x_{r,\theta})_{r\in R, \theta\in\mathcal{T}}$, such that $0 \leq x_{r,\theta} \leq u_{r,\theta}$ for all $r \in R$ and $\theta \in \mathcal{T}$, on the transport network in such a way that total capacity costs are minimized, provided the optimal adaptive strategy using only up to booked capacities $\vec{x}$ guarantees an expected number of containers delivered earlier than T of at least ρ units.

By setting a suitable state space $\mathcal{S}(\vec{x})$, action space $\mathcal{A}(\vec{x})$, reward function $\mathrm{Rew}(\cdot)$, transition probabilities and strategy space $\Pi^{\mathrm{MD}}(\vec{x})$ depending on the capacity selection $\vec{x}$, the adaptive problem can be formulated as a MDP and the resulting bilevel program can be

[1] Under the same conditions multiple deadlines and multiple destinations can be captured, provided containers have the same origin. Moreover, only identical containers are considered.

written as in (1).

We solve this problem by obtaining an LP-formulation of the MDP. Given the LP-formulation, we deal with the enormous decision space by looking for structural properties of the optimal strategy on specific classes of graphs. In order to establish the value of the synchromodal transport, we benchmark against a non-adaptive planning approach that will be considered under several settings characterized by different levels of a-priori capacity allocations.

$$
\begin{aligned}
\min \sum_{r\in R}\sum_{\theta\in\mathcal{T}} c_r x_{r,\theta} \qquad (1)\\
\text{s.t. } & 0 \le x_{r,\theta} \le u_{r,\theta} && \forall r\in R, \forall\theta\in\mathcal{T}\\
& x_{r,\theta}\in\mathbb{N}_{\ge 0} && \forall r\in R, \forall\theta\in\mathcal{T}\\
& y \ge \kappa\\
& y = \max_{\pi\in\Pi^{\text{MD}}} \mathbb{E}\left[\sum_{i=0}^{T-1}\text{Rew}(X_i, d_i(X_i)) + \text{Rew}^T(X_T)|X_0 = \underline{s}\right]\\
& \quad\text{s.t. } \mathcal{S} = \mathcal{S}(\vec{x})\\
& \quad\quad \mathcal{A} = \mathcal{A}(\vec{x})\\
& \quad\quad \Pi^{\text{MD}} = \Pi^{\text{MD}}(\vec{x})
\end{aligned}
$$

where $\underline{s}$ is the starting state with containers in node s, $\pi \in \Pi^{\text{MD}}$ is the list of decision rules $\pi = (d_1, \ldots, d_{T-1})$ to be taken ($d_i : \mathcal{S} \to \mathcal{A}$) and $\{X_i\}_{i=0}^T$ is the Markov process.

Keywords: *synchromodality, container transport, adaptive flow problem*

References

[1] B. Behdani, Y. Fan, B. Wiegmans, and R. Zuidwijk. Multimodal schedule design for synchromodal freight transport systems. *European Journal of Transport and Infrastructure Research*, 16(3):424–444, 2016.

[2] B. Colson, P. Marcotte, and G. Savard. An overview of bilevel optimization. *Annals of Operations Research*, 153(1):235–256, jun 2007.

[3] R. W. Hall. The Fastest Path through a Network with Random Time-Dependent Travel Times. *Transportation Science*, 20(3):182–188, aug 1986.

[4] E. D. Miller-Hooks and H. S. Mahmassani. Least Expected Time Paths in Stochastic, Time-Varying Transportation Networks. *Transportation Science*, 34(2):198–215, 2000.

[5] T. Rambha, S. D. Boyles, and S. T. Waller. Adaptive Transit Routing in Stochastic Time-Dependent Networks. *Transportation Science*, 50(3):1043–1059, 2016.

[6] P. M. Singh, M. J. van Sinderen, and R. J. Wieringa. Synchromodal Transport: Prerequisites, Activities and Effects. *ILS Conference 1-4 June 2016, Bordeaux, France*, 2016.

[7] B. van Riessen, R. R. Negenborn, and R. Dekker. Real-time container transport planning with decision trees based on offline obtained optimal solutions. *Decision Support Systems*, 89(March):1–16, 2016.

Combining Network Design and Graph Partitioning in a Multilevel Framework for Electricity Markets

Martin Schmidt*
Chair of EDOM, FAU Erlangen-Nürnberg, mar.schmidt@fau.de

Thomas Kleinert
Chair of EDOM, FAU Erlangen-Nürnberg, thomas.kleinert@fau.de

Keywords: *Multilevel optimization, Network design, Graph partitioning*

Abstract Network design and graph partitioning both are very important topics of discrete optimization with many different applications. In electricity market design, these problems often appear as ingredients in multilevel optimization problems. We consider a situation consisting of the following agents: a regulated transmission system operator (TSO), electricity supplying firms, and electricity consumers. First, the regulated TSO decides on the expansion of the current electricity transmission network and partitions the market area into different price zones. Its overall goal is to maximize the total social welfare. Afterward, taking the zoning of the market area and the network expansion decision of the regulator as given, the supplying firms decide on investment in further capacity and trade, together with the electricity consumers, on a series of spot markets. These spot markets are arranged in a way that only those physical constraints are taken into account that are defined on the cut edges of the transmission network, where the cut is given by the partitioning of the market area determined by the regulator. Hence, many physical constraints are ignored upon spot-market trading and the trading outcome might be infeasible with respect to all physical and technical transmission constraints. Lastly, potential infeasibilities are resolved in a redispatch stage, where the regulator modifies the production of certain electricity suppliers such that the modified quantities can be transported through the, possibly expanded, transmission network. The overall timing and structure of this trilevel problem is depicted in Figure 1. It is an extension of the model discussed in [1] and captures the main aspects of electricity trade in Germany.

The overall problem is an extremely challenging multilevel problem as it is often the case in energy market design optimization [2]. Our model is made up of three interconnected levels. The first level contains a network design as well as a graph partitioning (with additional connectivity constraints) problem and is modeled as a mixed-integer quadratic optimization problem (MIQP). The second level is again an MIQP that models spot-market interaction, which is constrained by the mixed-integer decisions of the first level. Finally, the third level is also an MIQP modeling a DC power flow problem with a welfare objective. It depends on the integer network design decisions of the first level.

From a mathematical point of view, the structure of the problem is given as in Figure 2. In general, and despite the recent algorithmic advances for solving bilevel problems like in [3], such a quadratic mixed-integer trilevel problem is not tractable. Fortunately, our model contains some specific structure that can be exploited in different algorithmic ways: All lower

max total social welfare (regulator)
s.t. network design
graph partitioning with connectivity constraints
max profits (competitive firms)
s.t. generation capacity investment,
production & demand constraints,
Kirchhoff's 1st law (zonal),
flow restrictions (inter-zonal)
min redispatch costs (TSO)
s.t. production & demand constraints,
lossless DC power flow constraints

Figure 1: Structure of the trilevel market model

$$
\begin{aligned}
\max \quad & \psi_1(W_2, W_3, X_1) \\
\text{s.t.} \quad & (W_1, X_1) \in \Omega_1 \\
& \quad \max \quad \psi_2(W_2) \\
& \quad \text{s.t.} \quad (W_2, X_1) \in \Omega_2 \\
& \quad\quad \min \quad \psi_3(W_2, W_3) \\
& \quad\quad \text{s.t.} \quad (W_2, W_3, X_1) \in \Omega_3
\end{aligned}
$$

Figure 2: Mathematical structure of the trilevel market model

level problems only depend on the decisions of the upper levels except for the first level's objective function that couples all three levels.

We present two different solutions approaches that both yield globally optimal solutions of the trilevel model. First, we prove that the given trilevel model can be reformulated as an equivalent bilevel problem. This bilevel problem consists of two MIQP levels, where the second level integers all are genuine first-level variables. Thus, a standard KKT reformulation approach can be applied and we further present dual bound tightening techniques that yield improved QP relaxations of the underlying single-level MIQP. Second, we present a problem-tailored Benders decomposition approach that also exploits the above mentioned specific structure of the problem. We derive Benders optimality cuts, which are then used to prove the overall correctness of the method.

Finally, we compare the performance and reliability of both algorithmic approaches and show that the problem-tailored Benders decomposition clearly outperforms the KKT reformulation technique. The Benders decomposition approach is then used for solving the trilevel model for instances of the German electricity market and we draw some conclusions regarding the German electricity market design.

References

[1] Grimm, V., Kleinert, T., Liers, F., Schmidt, M. and Zöttl, G., *Optimal price zones of electricity markets: A mixed-integer multilevel model and global solution approaches*, Optimization Methods and Software, 2017.

[2] Gabriel, S., Conejo, A., Fuller, J., Hobbs, B. and Ruiz, C. *Complementarity modeling in energy markets*, Springer Science & Business Media, Vol. 180, 2012.

[3] Fischetti, M., Ljubić, I., Monaci, M., and Sinnl, M., *A new general-purpose algorithm for mixed-integer bilevel linear programs*, Operations Research, Vol. 65-6, 2017.

Applications of Logic Constrained Equilibria to Traffic Networks and to Power Systems with Storage

Miguel F. Anjos*
NSERC-Hydro-Québec-Schneider Electric Industrial Research Chair, Inria International Chair, Polytechnique Montréal & GERAD, `anjos@stanfordalumni.org`

Franklin Djeumou Fomeni
Centre for Transport and Logistics (CENTRAL), Lancaster University Management School, `franklin@aims.ac.za`

Steven A. Gabriel
University of Maryland & GERAD, `sgabriel@umd.edu`

Abstract We study equilibria in traffic networks and in power system networks with storage in the presence of logic constraints. These constraints consist of binary variables that are added to complementarity-based equilibrium models. Although these models have been thoroughly studied, the addition of logic constraints can provide additional benefits for practical applications. We demonstrate that logic constraints can render classical equilibrium models more realistic by allowing the inclusion of useful features such as equity in network flows or threshold events. Specifically, for the traffic equilibrium problem, we show how logic constraints can introduce some equity in the assignment of traffic when more than one equilibrium exists. For power system networks, we show that the presence of a storage operator acting as a service provider will not only support the operation of a power grid, but will also help stabilize the price of electricity and avoid the well-documented price-shifting effect. Unlike previous works, our model considers the storage operator as a service provider rather than a competitor to the producers. We also consider the minimum power output of production. We present results illustrating the expanded capabilities and insights provided by these new paradigms.

Keywords: *Mixed linear complementarity problems, mixed integer linear optimization, traffic equilibrium, energy storage, power markets.*

References

[1] Djeumou Fomeni, F., S.A. Gabriel, and M.F. Anjos. *An RLT Approach for Solving the Binary-Constrained Mixed Linear Complementarity Problem*, Cahier du GERAD G-2015-60.

[2] Djeumou Fomeni, F., S.A. Gabriel, and M.F. Anjos. *Applications of Logic Constrained Equilibria to Traffic Networks and to Power Systems with Storage*, Cahier du GERAD G-2016-123.

A Tri-Level Energy Market Model

Léonard von Niederhäusern*
Team Inocs, Inria Lille-Nord Europe , leonard.vonniederhausern@inria.fr

Didier Aussel
PROMES-CNRS, UPVD, aussel@univ-perp.fr

Luce Brotcorne
Team Inocs, Inria Lille-Nord Europe, luce.brotcorne@inria.fr

Sébastien Lepaul
OSIRIS, EDF R&D, sebastien.lepaul@edf.fr

Abstract The energy domain faces multiple challenges, with economical, ecological and political interests at stake. To ensure the supply-demand balance, the solution originally consisted in fitting the production to the demand. However, alongside the rise of the smart-grid paradigm and the increasing importance of distributed generation, a new technique appeared : demand-side management (DSM), which consists in fitting the demand to the production. One of the tools to achieve DSM is time-of-use pricing : according to the time of consumption, the electricity consumers will be charged differently.

Our model considers four types of actors. At the upper level, an electricity furnisher sells the energy it produces/buys on the spot market. At an intermediary level, two types of actors buy energy to the furnisher for consumption : so called local agents use the bought energy to power their own devices (and thus decide on their consumption schedule), whereas aggregators transmit the energy they buy to end-users with whom they are in contract. These end-users constitute the third level of our model. The interaction between aggregators and the corresponding end-users consist in rewards offered by the aggregator to its end-users to incur load shifting, as suggested in [1]. Furthermore, our model offers the possibility for actors of the second level to exchange energy among themselves.

Since bilevel programming is already difficult to solve in general, solving a tri-level program requires particularly careful handling. The first step of the resolution consists in reducing the tri-level model to a bi-level one. This is achieved thanks to an explicit formula for the end-users' programs. Then a characterization of the Nash equilibrium among the actors of the intermediary level leads to two possible reformulations : one for the optimistic case, and another one for a "semi-optimistic" (or "semi-pessimistic") case, that might be more realistic. These two reformulations allow a quick and efficient resolution with commercial solvers. To illustrate this, we conclude our talk with numerical results.

Keywords: *tri-level programming, demand side management, energy markets*

References

[1] Gkatzikis, L., Koutsopoulos, I. and T. Salonidis, *The Role of Aggregators in Smart Grid Demand Response Markets*, IEEE Journal on Selected Areas in Communications. **31(7)** (2013), 1247–1257.

Unit Commitment under Market Equilibrium Constraints

Bernard Fortz*
Department of Computer Sciences, Université libre de Bruxelles and INOCS, INRIA, bernard.fortz@ulb.ac.be

Luce Brotcorne
INOCS, INRIA, luce.brotcorne@inria.fr

Fabio D'Andreagiovanni
Université de Technologie de Compiègne, f.dandreagiovanni@gmail.com

Jérôme De Boeck
Department of Computer Sciences, Université libre de Bruxelles and INOCS, INRIA, jerome.de.boeck@ulb.ac.be

Abstract The classical Unit Commitment problem (UC) can be essentially described as the problem of establishing the energy output of a set of generation units over a time horizon, in order to satisfy a demand for energy, while minimizing the cost of generation and respecting technological restrictions of the units (e.g., minimum on/off times, ramp up/down constraints). The UC is typically modelled as a (large scale) mixed integer program and its deterministic version, namely the version not considering the presence of uncertain data, has been object of wide theoretical and applied studies over the years.

Traditional (deterministic) models for the UC assume that the net demand for each period is perfectly known in advance, or in more recent and more realistic approaches, that a set of possible demand scenarios is known (leading to stochastic or robust optimization problems).

However, in practice, the demand is dictated by the amounts that can be sold by the producer at given prices on the day-ahead market. One difficulty therefore arises if the optimal production dictated by the optimal solution to the UC problem cannot be sold at the producer's desired price on the market, leading to a possible loss. Another strategy could be to bid for additional quantities at a higher price to increase profit, but that could lead to infeasibilities in the production plan.

Our aim is to model and solve the UC problem with a second level of decisions ensuring that the produced quantities are cleared at market equilibrium. In their simplest form, market equilibrium constraints are equivalent to the first-order optimality conditions of a linear program. The UC in contrast is usually a mixed-integer nonlinear program (MINLP), that is linearized and solved with traditional Mixed Integer (linear) Programming (MIP) solvers. Taking a similar approach, we are faced to a bilevel optimization problem where the first level is a MIP and the second level linear.

In this talk, as a first approach to the problem, we assume that demand curves and offers of competitors in the market are known to the operator. This is a very strong and unrealistic hypothesis, but necessary to develop a first model. Following the classical approach for these models, we present the transformation of the problem into a single-level program by rewriting

and linearizing the first-order optimality conditions of the second level. Then we present some preliminary results on the performance of MIP solvers on this model. Our future research will focus on strengthening the model using its structure to include new valid inequalities or to propose alternative extended formulations, and then study a stochastic version of the problem where demand curves are uncertain.

Keywords: *Unit commitment, market equilibrium, day-ahead market, bilevel optimization*

A polynomial algorithm for continuous bilevel knapsack problems

Margarida Carvalho*
Canada Excellence Research Chair, Polytechnique de Montréal, margarida.carvalho@polymtl.ca
Andrea Lodi
Canada Excellence Research Chair, Polytechnique de Montréal, andrea.lodi@polymtl.ca
Patrice Marcotte
Département d'informatique et de recherche opérationnelle, Université de Montréal, marcotte@iro.umontreal.ca

Abstract

In general, bilevel programs are a mathematical challenging class of problems. In linear bilevel programs (BLPs), all decision variables are continuous, upper-level and lower-level objective functions and constraints are linear. Jeroslow [5] proved that even BLPs are NP-hard, *i.e.,* there is no hope of building a polynomial time algorithm to solve a BLP if $NP \neq P$ (a highly likely event).

In this work we analyse BLPs where the lower level is modelled as a continuous knapsack problem. In particular, first the leader makes his/her decision and fixes the values of his/her variables, and afterwards the follower reacts by solving a continuous knapsack problem. The follower's knapsack depends on the leader's strategy. To start, we analyse the case in which the leader can decide the fraction of each item available for the follower and the leader aims to minimize the total value of the items packed by the follower. This BLP is based on the binary BLP tackled in [4] and [2]. This can be formulated as follows:

$$\min_{(x,y)\in[0,1]^n\times[0,1]^n} \sum_{i=1}^{n} p_i y_i \tag{1a}$$

$$\text{subject to} \quad \sum_{i=1}^{n} v_i x_i \leq C_u \tag{1b}$$

where $y_1, \dots, y_n$ solves the follower's problem

$$\max_{y\in[0,1]^n} \sum_{i=1}^{n} p_i y_i \quad \text{s.t.} \quad \sum_{i=1}^{n} w_i y_i \leq C_l \quad \text{and} \tag{1c}$$

$$y_i \leq 1 - x_i \quad \text{for } 1 \leq i \leq n. \tag{1d}$$

We will also extend our study to more general formulations of the leader's objective function. Furthermore, we will discuss how to adapt our approach for the case in which the

leader decides the follower's knapsack capacity, instead of interdicting items. The latter case is motivated by the binary bilevel knapsack programs studied in [3], [6] and [1].

Through this work, we assume that the leader has perfect knowledge of the follower's scenario (objective function and constraints) and also of the follower's behaviour. Furthermore, we assume that the follower can fully observe the leader's action. In order to have a bilevel program, the leader's objective function or constraints must depend on the follower's decision. In our work, the leader's objective function takes the follower's reaction into account. We remark that for Problem (1) there is no need to consider the optimistic and pessimistic cases.

In order to solve Problem (1), we apply a inverse optimization approach. To that end, we make use of the well-known result [7] for the computational of optimal solutions to a continuous knapsack problem. This leads to the conclusion that it is sufficient to determine the follower's item that in the optimal solution is *critical*. We prove that this can be done in polynomial time. Furthermore, we highlight how our algorithmic idea can be extend to more general models.

Keywords: *Bilevel programming, Continuous knapsack problem, Polynomial time, Inverse optimization*

References

[1] Brotcorne, L., Hana, S., and Mansi, R., *One-level reformulation of the bilevel Knapsack problem using dynamic programming*, Discrete Optimization. **10** (2013), 1–10.

[2] Caprara, A., Carvalho, M., Lodi, A., and Woeginger, G.J., *Bilevel knapsack with interdiction constraints.*, INFORMS Journal on Computing. **28** (2016), 319–333.

[3] Dempe, S., and Richter, K., *Bilevel programming with knapsack constraints*, Central European Journal of Operations Research. **8** (2000), 93–107.

[4] DeNegre, S., *Interdiction and discrete bilevel linear programming.*, Ph.D. dissertation, Lehigh University. (2011).

[5] Jeroslow, R. G., *The polynomial hierarchy and a simple model for competitive analysis*, Mathematical Programming. **32** (1985), 146–164.

[6] Mansi, R., Alves, C., de Carvalho, J. M. V., Hana, S., *An exact algorithm for bilevel 0-1 knapsack problems*, Mathematical Problems in Engineering. (2012).

[7] Martello, S., and Toth, P., *Knapsack problems: algorithms and computer implementations.*, John Wiley & Sons, Inc., New York, (1990).

Contributed talk

Newton method for bilevel optimization: Theory and extensive numerical experiments

Alain B. Zemkoho*
School of Mathematics, University of Southampton, Southampton, UK,
a.b.zemkoho@soton.ac.uk

Joint work with Shenglong Zhou (University of Southampton, UK) and Andreas Fischer (Technical University of Dresden, Germany)

Abstract Considering the standard optimistic bilevel optimization problem, there are two main approaches to reformulate it as a single-level optimization problem. On the one hand, we have the *KKT (i.e., Karush-Kuhn-Tucker) refomulation*, strongly connected to the class of mathematical programs with *equilibrium* (or *complementarity*) constraints (MPECs or MPCCs for short). We will not be pursuing this approach in this talk, but instead, we will focus our attention on *lower-level value function* (LLVF) reformulation, which is typically nondifferentiable. Also, even if all the functions involved in the problem are fully convex, the feasible set is still nonconvex. A number of publications on solution algorithms for bilevel optimization problems based on the LLVF reformulation or a closely related transformation have been published recently, including global optimization techniques and local optimization-types. The aim of the work to be presented at this workshop is to compute stationary points for the optimistic bilevel optimization while using the LLVF reformulation; precisely, we compute stationary points which can be locally optimal under suitable conditions, also discussed. But unlike in some previous works, our approach is based on techniques which allow us to avoid to directly compute the value function. The contribution of the work to be presented, cf. [2] and [3], is five-fold:

1. To develop a tractable Newton method to solve the aforementioned LLVF reformulation, we first introduce a new stationarity concept and established useful relationships with known ones.

2. For the first time in the literature, we develop second order sufficient optimality conditions for bilevel optimization based on the LLVF reformulation.

3. We then construct a Newton method with corresponding convergence results, which are based in part on the condition that the Hessian matrix of the *Lagrangian function of the upper-level player* be larger than that of the lower-level player on a certain cone of feasible directions.

4. More importantly, our method converges on all the nonlinear problems in BOLIB [1]; i.e.; 126 problems in total. Also, for 117 for which a true solution is known or at least there is a best known solution, we either achieve this value or a better one. It might be useful to recall that no method designed for nonlinear bilevel optimization has been tested on such a number problems.

5. Since a parameter λ was introduced by Ye and Zhu in 1995 in the solution process of the LLVF reformulation via the *partial calmness* concept, no experimental study has been done to illustrate how it can be selected. We provide a benchmark study in this paper on the selection of λ.

Keywords: *Bilevel optimization, Newton method*

References

[1] Tin, A., and A.B. Zemkoho, and S. Zhou, *BOLIB: Bilevel OPtimization LIBrary of Test Problems and Solvers*, Technical Report, School of Mathematics, University of Southampton, Southampton, UK (under preparation)

[2] Zemkoho, A.B., and S. Zhou, A. Fischer *Newton method for bilevel optimization: Theory and extensive numerical experiments*, Technical Report, School of Mathematics, University of Southampton, Southampton, UK (under preparation)

[3] Zemkoho, A.B., and S. Zhou, A. Fischer *Numerical Experiments for the paper "Newton method for bilevel optimization: Theory and extensive numerical experiments"*, Technical Report, School of Mathematics, University of Southampton, Southampton, UK (under preparation)

Finding Optimistic Solutions in Quadratic Bilevel Optimization Problems

Andrei Orlov*
Matrosov Institute for System Dynamics & Control Theory SB RAS, anor@icc.ru

Abstract This work addresses quadratic bilevel optimization problems in their optimistic statement. The reduction of the bilevel problem to a series of nonconvex mathematical optimization problems, together with the specialized Global Search Theory, is used for developing methods of local and global searches to find optimistic solutions. Numerical testing of the method on specially constructed instances demonstrated the efficiency of the approach.

Keywords: *quadratic bilevel optimization, optimistic solution, KKT-approach, penalty approach, global search theory, computational simulation*

Consider the following quadratic-quadratic problem of bilevel optimization in its optimistic statement. In this case, according to the theory (see [2] and references in it), at the upper level we perform the minimization with respect to the variables of both levels which are in cooperation:

$$\left.\begin{array}{c} F(x,y) := \dfrac{1}{2}\langle x, Cx\rangle + \langle c, x\rangle + \dfrac{1}{2}\langle y, Dy\rangle + \langle d, y\rangle \downarrow \min\limits_{x,y}, \\ x \in X := \{x \in I\!R^m | Ax \le b\}, \\ y \in Y_*(x) := \operatorname{Arg}\min\limits_{y}\{\dfrac{1}{2}\langle y, D_1 y\rangle + \langle d_1, y\rangle + \langle x, Qy\rangle \mid y \in Y(x)\}, \\ Y(x) \stackrel{\triangle}{=} \{y \in I\!R^n | A_1 x + B_1 y \le b_1\}, \end{array}\right\} \qquad (\mathcal{QBP})$$

where $A \in I\!R^{p\times m}$, $A_1 \in I\!R^{q\times m}$, $B_1 \in I\!R^{q\times n}$, $C \in I\!R^{m\times m}$, $D, D_1 \in I\!R^{n\times n}$, $Q \in I\!R^{m\times n}$, $c \in I\!R^m$, $d, d_1 \in I\!R^n$, $b \in I\!R^p$, $b_1 \in I\!R^q$. Additionally, $C = C^T \ge 0$, $D = D^T \ge 0$, $D_1 = D_1^T \ge 0$.

According to the well known KKT-approach and penalty approach we can reduce Problem $(\mathcal{QBP})$ to the following σ-parametrized nonconvex problem with a convex feasible set [2]:

$$\left.\begin{array}{c} \Phi(x,y,v) := \dfrac{1}{2}\langle x, Cx\rangle + \langle c, x\rangle + \dfrac{1}{2}\langle y, Dy\rangle + \langle d, y\rangle + \\ +\sigma\,\langle v, b_1 - A_1 x - B_1 y\rangle \downarrow \min\limits_{x,y,v}, \quad (x,y,v) \in D, \end{array}\right\} \qquad (\mathcal{DC}(\sigma))$$

where $\sigma > 0$ is a penalty parameter, v is the vector of Lagrange multipliers for the lower level problem and $D := \{(x,y,v) \mid Ax \le b,\ \ D_1 y + d_1 + xQ + vB_1 = 0,\ \ v \ge 0,\ \ A_1 x + B_1 y \le b_1\}$.

Note that for a fixed σ problem $(\mathcal{DC}(\sigma))$ has a bilinear structure and belongs to the class of d.c. minimization problems [3] with a convex feasible set. It is easy to see that the objective function of $(\mathcal{DC}(\sigma))$ can be represented as a difference of two convex functions [3].

We will employ, for example, the following d.c. representation based on the known property of a scalar product:

$$\Phi(x,y,v) = g(x,y,v) - h(x,y,v), \qquad (1)$$

where $g(x,y,v) = \frac{1}{2}\langle x, Cx\rangle + \langle c, x\rangle + \frac{1}{2}\langle y, Dy\rangle + \langle d, y\rangle + \sigma\langle b_1, v\rangle + \frac{\sigma}{4}\|A_1x - v\|^2 + \frac{\sigma}{4}\|B_1y - v\|^2$, $h(x,y,v) = \frac{\sigma}{4}\|A_1x + v\|^2 + \frac{\sigma}{4}\|B_1y + v\|^2$. Note that the so-called *basic nonconvexity* in Problem $(\mathcal{DC}(\sigma))$ is provided by the function h (for more details, refer to [2], [3]).

So we can apply the Global Search Theory in d.c. minimization problems [3] to seek a global solution to Problem $(\mathcal{DC}(\sigma))$ with a fixed σ. Within that theory, it is first required to construct a local search procedure that takes into consideration special features of the problem under study.

It is noteworthy that nonconvexity in Problem $(\mathcal{DC}(\sigma))$ is generated by only a bilinear component $\langle v, b_1 - A_1x - B_1y\rangle$. On the basis of this fact, we suggest to perform the local search in Problem $(\mathcal{DC}(\sigma))$ using the idea of the successive solution by different groups of variables. Earlier this idea was successfully applied in solving bimatrix games, problems of bilinear programming, and quadratic-linear bilevel problems (see [2], [3] and references in them). Therefore, a specialized local search method appears.

Let there be given a starting point (x_0, y_0, v_0).

V-procedure

Step 0. Set $s := 1$, $v^s := v_0$.
Step 1. Using a suitable quadratic programming method, find the $\frac{\rho_s}{2}$-solution (x^{s+1}, y^{s+1}) to the problem

$$\left.\begin{array}{c} \frac{1}{2}\langle x, Cx\rangle + \langle c, x\rangle + \frac{1}{2}\langle y, Dy\rangle + \langle d, y\rangle - \sigma\langle v^s A_1, x\rangle - \sigma\langle v^s B_1, y\rangle \downarrow \min\limits_{x,y}, \\ Ax \le b, \quad A_1x + B_1y \le b_1, \quad D_1y + d_1 + xQ + v^s B_1 = 0. \end{array}\right\} \qquad (\mathcal{QP}(v^s))$$

Step 2. Find the $\frac{\rho_s}{2}$-solution v^{s+1} to the following LP-problem:

$$\left.\begin{array}{c} \langle b_1 - A_1x^{s+1} - B_1y^{s+1}, v\rangle \downarrow \min\limits_{v}, \\ D_1y^{s+1} + d_1 + x^{s+1}Q + vB_1 = 0, \quad v \ge 0. \end{array}\right\} \qquad (\mathcal{LP}(x^{s+1}, y^{s+1}))$$

Step 3. Set $s := s + 1$ and move to **Step 1.**

We can prove the convergence theorem for the V-procedure to a critical point with special properties [2], and we can introduce the stopping criteria to obtain an approximately critical point. As well-known, the local search does not provide, in general, a global solution in nonconvex problems of even moderate dimension [3]. Therefore, in what follows we discuss the procedure of escaping critical points obtained during the local search. The procedure is based on the Global Optimality Conditions (GOCs) developed by A.S. Strekalovsky for the d.c. minimization problems [3], and has the following steps.

Suppose we know some approximately critical point (x^k, y^k, z^k) in Problem $(\mathcal{DC}(\sigma))$ with the goal function value $\zeta_k := \Phi(x^k, y^k, v^k)$, obtaining by the V-procedure. Then we perform the following chain of operations.

1) Choose a number $\gamma \in [\gamma_-, \gamma_+]$, where $\gamma_- := \inf(g, D)$, $\gamma_+ := \sup(g, D)$. We can take, for example, $g(x^k, y^k, z^k)$ as a starting value of the parameter γ [3].

2) Furtherfore, construct some finite approximation

$$\mathcal{A}_k = \{(z^i, u^i, w^i) \mid h(z^i, u^i, w^i) = \gamma - \zeta_k, \ \ i = 1, ..., N_k\}$$

for the level surface $U(\zeta_k) = \{(x, y, v) \mid h(x, y, v) = \gamma - \zeta_k\}$ of the convex function $h(\cdot)$.

3) For all approximation points $\mathcal{A}_k$ verify the inequality $g(z^i, u^i, w^i) \leq \gamma,\ i = 1, 2, ..., N_k$, that follows from the global optimality conditions for Problem $(DC(\sigma))$ [3]. If this inequality is satisfied, then the approximation point will be used in process. Otherwise, the point (z^i, u^i, w^i) is useless, because it is not able to improve the current point [3].

4) For each point $(z^i, u^i, w^i),\ i \in \{1, 2, ..., N_k\}$ chosen at Stage 3) find approximate solutions $(\bar{z}^i, \bar{u}^i, \bar{w}^i)$ of the linearized (with respect to the basic nonconvexity) problems:

$$g(x, y, v) - \big\langle \nabla h(z^i, u^i, w^i), (x, y, v) \big\rangle \downarrow \min_{x,y,v},\ \ (x, y, v) \in D. \qquad (\mathcal{PL}(z^i, u^i, w^i))$$

5) Using the points $(\bar{z}^i, \bar{u}^i, \bar{w}^i)$, perform an additional local search that delivers approximately critical points $(\hat{x}^i, \hat{y}^i, \hat{v}^i),\ i \in \{1, ..., N\}$ in Problem $(\mathcal{DC}(\sigma))$.

6) For the chosen $i \in \{1, \ldots, N_k\}$, solve the level problem:

$$\left.\begin{array}{c} \langle \nabla_x h(z, u, w), \hat{x}^i - z \rangle + \langle \nabla_y h(z, u, w), \hat{y}^i - u \rangle + \\ + \langle \nabla_v h(z, u, w), \hat{v}^i - w \rangle \uparrow \max\limits_{(z,u,w)}, \qquad h(z, u, w) = \gamma - \zeta_k. \end{array}\right\} \qquad (\mathcal{U}_i)$$

Note that definition for $h(\cdot)$ makes it possible to solve Problem $(\mathcal{U}_i)$ analytically. Let (z_0^i, u_0^i, w_0^i) be the approximate solution to this problem.

7) If for some $j \in \{1, \ldots, N_k\}$ the following inequality holds

$$\begin{array}{c} g(\hat{x}^j, \hat{y}^j, \hat{v}^j) - \gamma < \langle \nabla_x h(z_0^j, u_0^j, w_0^j), \hat{x}^j - z_0^j \rangle + \\ + \langle \nabla_y h(z_0^j, u_0^j, w_0^j), \hat{y}^j - u_0^j \rangle + \langle \nabla_v h(z_0^j, u_0^j, w_0^j), \hat{v}^j - w_0^j \rangle, \end{array}$$

then, due to convexity of $h(\cdot)$, we obtain

$$\gamma - h(z_0^j, u_0^j, w_0^j) = \zeta_k = \Phi(x^k, y^k, v^k) > \Phi(\hat{x}^j, \hat{y}^j, \hat{v}^j).$$

Thus, we constructed the point $(\hat{x}^j, \hat{y}^j, \hat{v}^j) \in D$, which is better than (x^k, y^k, v^k). If we failed to improve the value of ζ_k using all approximation points $\mathcal{A}_k$, then we have to continue the one-dimensional search with respect to γ on the segment $[\gamma_-, \gamma_+]$.

As a result, taking into account the features of Problem $(\mathcal{DC}(\sigma))$ and basing on the stages of the global search 1)-7), we have constructed and implemented the Global Search Algorithm (GSA) in the quadratic bilevel problems.

Testing the GSA we performed by using the special method for generation of bilevel test cases proposed in [1]. The idea of such generation is based on constructing bilevel problems of an arbitrary dimension with the help of the so-called kernel problems, which are one-dimensional bilevel problems with known local and global solutions. The computational simulation shows the efficiency of developed approach to quadratic bilevel optimization problems of high dimension.

References

[1] Calamai, P., and L. Vicente *Generating Quadratic Bilevel Programming Test Problems*, ACM Transaction on Math. Software. **20** (1994), 103–119.

[2] Orlov, A.V. *A Nonconvex Optimization Approach to Quadratic Bilevel Problems*, In: R. Battiti, D. Kvasov, Y. Sergeyev (eds.) "Learning and Intelligent Optimization Conference LION11, Nizhny Novgorod, Russia", LNCS **10556**, Springer, 2017, 222–234.

[3] Strekalovsky, A.S. *On solving optimization problems with hidden nonconvex structures*, In T.M. Rassias, C.A. Floudas, and S. Butenko (eds.) "Optimization in science and engineering", Springer, 2014, 465–502.

Computing a Pessimistic Leader-Follower Equilibrium with Multiple Followers: the Mixed-Pure Case

Stefano Coniglio*
Department of Mathematical Sciences, University of Southampton,
s.coniglio@soton.ac.uk

Nicola Gatti
Department of Electronics, Computer Science, and Biotechnology, Politecnico di Milano,
nicola.gatti@polimi.it

Alberto Marchesi
Department of Electronics, Computer Science, and Biotechnology, Politecnico di Milano,
alberto.marchesi@polimi.it

Abstract The computing of equilibria in *leader-follower* (or *Stackelberg*) games maximizing a certain utility function (such as the social welfare) is a classical application of bilevel programming. In this area, the (search) problem of computing a leader-follower equilibrium has been widely investigated in the scientific literature in, almost exclusively, the single-follower setting [vSZ10]. LFGs encompass a broad array of real-world games. A prominent example is that one of security games, where a defender, acting as a leader, is tasked to allocate scarce resources to protect valuable targets from an attacker, acting as a follower [APS$^+$11, KJT$^+$09, PPM$^+$08]. Besides the security domain, applications can be found in, among others, interdiction games [CCLW16, MMO$^+$17], toll-setting problems [LV16], and network routing [ACCG13].

Although the *optimistic* and *pessimistic* versions of the problem, i.e., those where the single follower breaks any ties among multiple equilibria either in favor or against the leader, are solved with different methodologies, both cases allow for efficient, polynomial-time algorithms based on linear programming. The situation is quite different when multiple followers are present, a case in which results are only sporadic and strictly depend on the nature of the followers' game.

We investigate, in this talk, the setting of normal-form games with a single leader and multiple followers who, after observing the leader's commitment to a mixed-strategy, play a Nash equilibrium in the resulting game. The corresponding search problem is, as we show, not in Poly-APX unless P = NP both in the optimistic and pessimistic versions. In particular, exact algorithms are known only for the optimistic case.

We focus, here, on the case where the followers play pure strategies—a restriction that applies to a number of real-world scenarios—under the assumption of pessimism (as we show, the optimistic version of the problem can be straightforwardly solved in polynomial time). After casting this search problem as a *pessimistic bilevel programming problem*, we show that, with two followers, the problem is NP-hard and, with three or more followers, it is not in Poly-APX unless P = NP. This last result matches the inapproximability result which holds for

the unrestricted case (where both leader and followers are allowed to play a mixed strategy) and shows that, differently from what happens in the optimistic version, hardness in the pessimistic problem is not due to the adoption of mixed strategies.

We then show that the problem admits, in the general case, a supremum but not a maximum (as it is often the case of pessimistic bilevel programming problems). By relying on a duality argument, we construct an exact single-level mathematical programming reformulation of the (pessimistic) problem, calling for the maximization of a nonconcave quadratic function over an unbounded nonconvex feasible region, the latter defined by linear and quadratic constraints. We show that, in the general case, this formulation admits optimal solutions over the extended reals but not over the reals (i.e., with some variables taking value $+\infty$) and, to mitigate the fact that, due to this, state-of-the-art solvers would not be capable of solving it to optimality, we introduce a restricted version in which all the variables are (artificially) bounded.

We propose an exact *ad hoc* algorithm for the (pessimistic) problem capable of solving it to optimality without restrictions—we also embed the algorithm within a branch-and-bound scheme capable of computing the supremum of the problem and, for cases where there is no leader's strategy where such value is attained (i.e., for the cases where the problem admits a supremum but not a maximum), also an α-approximate strategy where $\alpha > 0$ is any additive loss.

We conclude by evaluating the scalability of our algorithms via computational experiments on a well-established testbed of game instances.[1]

Keywords: *leader-follower (Stackelberg) games, equilibria computation, pessimistic bilevel programming*

References

[ACCG13] E. Amaldi, A. Capone, S. Coniglio, and L. G. Gianoli. Network optimization problems subject to max-min fair flow allocation. *IEEE COMMUN LETT*, 17(7):1463–1466, 2013.

[APS+11] B. An, J. Pita, E. Shieh, M. Tambe, C. Kiekintveld, and J. Marecki. Guards and Protect: Next generation applications of security games. *ACM SIGecom Exchanges*, 10(1):31–34, 2011.

[BCGM17] N. Basilico, S. Coniglio, N. Gatti, and A. Marchesi. Bilevel programming approaches to the computation of optimistic and pessimistic single-leader-multi-follower equilibria. In *16th International Symposium on Experimental Algorithms (SEA 2017)*, Leibniz International Proceedings in Informatics, pages 69:1–69:14. Schloss Dagstuhl—Leibniz-Zentrum fÃ¼r Informatik, Dagstuhl Publishing, Germany, 2017.

[CCLW16] A. Caprara, M. Carvalho, A. Lodi, and G. J. Woeginger. Bilevel knapsack with interdiction constraints. *INFORMS J COMPUT*, 28(2):319–333, 2016.

[CGM17] S. Coniglio, N. Gatti, and A. Marchesi. Pessimistic leader-follower equilibria with multiple followers. In *IJCAI*, pages 171–177. 2017.

[1]Preliminary portions of this work have appeared in [BCGM17, CGM17].

[KJT+09] C. Kiekintveld, M. Jain, J. Tsai, J. Pita, F. Ordóñez, and M. Tambe. Computing optimal randomized resource allocations for massive security games. In *AAMAS*, pages 689–696, 2009.

[LV16] M. Labbé and A. Violin. Bilevel programming and price setting problems. *ANN OPER RES*, 240(1):141–169, 2016.

[MMO+17] J. Matuschke, S. T. McCormick, G. Oriolo, B. Peis, and M. Skutella. Protection of flows under targeted attacks. *OPER RES LETT*, 45(1):53–59, 2017.

[PPM+08] P. Paruchuri, J. P. Pearce, J. Marecki, M. Tambe, F. Ordonez, and S. Kraus. Playing games for security: an efficient exact algorithm for solving bayesian stackelberg games. In *AAMAS*, pages 895–902, 2008.

[vSZ10] B. von Stengel and S. Zamir. Leadership games with convex strategy sets. *GAME ECON BEHAV*, 69:446–457, 2010.

An exact algorithm to solve a cloud services bi-level problem

Lilian Lopez-Vera *
Facultad de Ciencias Físico-Matemáticas, Universidad Autónoma de Nuevo León, lilian.llv@hotmail.com

José-Fernando Camacho-Vallejo
Facultad de Ciencias Físico-Matemáticas, Universidad Autónoma de Nuevo León, jose.camachovl@uanl.edu.mx

Luce Brotcorne
INRIA Lille Nord-Europe, Luce.Brotcorne@inria.fr

Abstract Recently cloud services have had a remarkable boom in real life. However, there are not many mathematical programming models yet that could help us analyze situations related to this topic. This is the reason that motivated us to study a situation in which it is necessary to set prices and decide the quantity that will be available for the service but without losing sight of customer satisfaction. The latter is measured by the quality of the service taking into account congestion functions. To model this problem, we propose a bi-level programming model with one leader and one follower. A global cost function regarding all the customers is considered at this stage of the research for optimizing the lower level problem. We propose an exact algorithm based on listing all the possible follower decisions for a fixed leader's one. Numerical experimentation conducted over a set of randomly created instances shows the convenience to use the proposed algorithm for solving this bi-level problem

Keywords: *Cloud Computing, Bi-Level Programming, Quality of Service*

1 Introduction

Cloud Computers provide on-demand resources and services over a network, with two different but related types of clouds, those that provide computing instances on demand and those that provide computing capacity on demand [1]. Both use similar machines, but the first is designed to scale out by providing additional computing instances, whereas the second is designed to support data -or compute- intensive applications via scaling capacity. According to [1], the current interest in focusing on cloud computing is due to three important differences: (i) Scale, some companies that rely on cloud computing have infrastructures that scale over several data centers, (ii) Simplicity, prior to cloud-based computing services, writing code for high performance and distributed computing was relatively complicated and usually required working with grid-based services, developing explicit code, and employing other specialized methods, and (iii) Pricing, cloud computing is often offered with a pricing model that lets you pay as you go and for just the services that you need. Analyzing the third characteristic, in [2] are studied the reasons why a customer may rent software in preference to buying it. The reasons are: (i) the software is for use in a short-term project, (ii) the customer may simply want to gain experience of using the software, (iii) the customer wants to test and evaluate the usability of the software, or (iv) the customer wants to avoid negative network externality. And it can be proved that these reasons are true by watching how the cloud services have gradually gained popularity in the last years.

Now, with a widely known and increasingly popular service, and focusing on the clouds providing computing instances on demand, we aim to set a price for it, and it is important to know the most common pricing scheme (pay as you go) employing that a unit of service is charged at a fixed price per unit of time is what we are aiming to find. But, is it better to establish a fixed price for a service rather than a spot price? Looking for similar services we found that there would always be unused resources in the virtual capacity of the cloud. Based on the ideas of establishing spot prices with the aim of minimizing the disutilities and maximizing the revenue presented in [3], our problem takes place. In a highly competent market, such as cloud servers, the customers' demand is closely related with the quality of the service (QoS), which is a very important feature in our case study and it is commonly ignored in other study cases.

2 Problem Statement

In the modeled situation we consider a company which tries to set a pricing scheme for its available resource, aiming to maximize its income. The company must take into account that if its resource is used by the customers, a delay will immediately affect the performance of the service. At the same time, customers attempt to minimize the global cost based on their consumption necessities and the company's pricing scheme.

Based on this idea, we propose a mathematical bi-level programming model which involves two decision levels. The upper level corresponds to the cloud company (leader) and the lower level associated with the customers (followers). By considering a leader's objective function aiming to maximize the profit, we are seeking to set a pricing policy, in which the leader must decide that price for different capacity levels of their resource in each time period. This leads to having an objective function represented by the total income minus a function that will show the strong link between demand and quality of service. The latter being translated into money by a delay factor which is increased according to the number of customers using the same service in the same period of time. It is convenient to remark that our function is bounded by the maximum price that a customer is willing to pay for the service.

For the followers, who pursue to minimize their global cost (which is simply identified as the sum of the price fixed in each period of time, multiplied by the amount contracted in that period) and based on the price policy set by the leader, they must decide the most convenient periods of time to hire the service and the amount they want to use in each period, where it must be assured that their global demand is fully covered and that they are being completely satisfied with just one contract per period of time.

3 Current Work

The exact method that we propose is based on [3]. The idea is to reflect the importance of decreasing the consumption peaks of the customers at each period of time, which will indirectly help us to improve the performance of the service by not having periods of time with overload. To achieve this, customers are being moved by the price, where based on each customer demand, the proposed algorithm will pre-compute all the set of possible solutions (a possible solution must take into consideration that a job may not be stopped and restarted in non-consecutive periods), and the follower must decide which set that satisfies its preferences also gives him the best deal.

References

[1] R. Grossman, 2009. The Case for Cloud Computing, Cloud Computing, 1520–9202/09, 23–27.

[2] Choudhary, V., Tomak, K., Chaturvedi, A., 1998. Economic benefits of renting software. J. Org. Comput. Electron. Commerce 8 (4), 277–305.

[3] Afsar, S., Brotcorne, L., Marcotte, P., Savard, G., 2016, Achieving an optimal trade-off between revenue and energy peak within a smart grid environment, Optimization Methods in Renewable Energy Systems

Single-minded bundle pricing problem: Exact methods based on mathematical programming.

Fränk Plein*
Department of Computer Science, Université Libre de Bruxelles, frank.plein@ulb.ac.be

Sourour Elloumi
Department of Applied Mathematics, ENSTA Paristech,
sourour.elloumi@ensta-paristech.fr

Martine Labbé
Department of Computer Science, Université Libre de Bruxelles, martine.labbe@ulb.ac.be

Abstract In a modern day world, where new technologies allow for complex decision-making processes, a retailing company must implement a product pricing framework, in order to remain competitive. In the present work, we analyse one specific problem in product pricing under a model of single-minded customer behaviour, determined by a bundle and a budget. A bundle is the subset of products, a given customer wants to purchase and is willing to pay at most his budget for. We define the total price of a bundle to be the sum of all prices of the products inside. The client's single-minded behaviour can be described as binary decision process. The bundle is purchased if and only if the total price is less than the budget. The objective of the company is to maximise its total profits. This problem is known under Single-Minded Bundle Pricing Problem (SMBPP) and has been introduced only recently in the literature of algorithmic pricing. In the scope of this work, we assume that the supply is unlimited, which would be an adequate assumption for digital goods and goods that can be restocked in negligible time. We also assume that both bundle and budget of the clients are part of the input. This eliminates the need to consider market competitors, as the budget can represent the price a customer would have to pay for the same products, but buying from a competitor. Assuming the bundles are known in advance is also non-restrictive, especially since in common situations they are predefined by the optimising company.

After a brief literature review on product pricing, we noted that most of the research on the SMBPP is dedicated to complexity results and approximability. We therefore contribute to a missing piece of literature of exact methods given by the resolution of mathematical programs. At first we propose a very intuitive approach to formulate the problem as a bi-level program using only non-negative price variables and the binary buying decision of each client. And show its equivalence to a single-level mixed-integer non-linear program (MINLP). The non-linearity arises from naturally appearing product terms of price variables and buying decisions. Applying the classical techniques of McCormick, we linearise this formulation in two different ways to obtain mixed-integer linear programs (MILP) that can be solved exactly

by using branch-and-bound methods. A first formulation is obtained by replacing the product of the total bundle price and the corresponding client's buying decision. A second model, on the other hand, replaces the product of individual products and buying decisions.

Referring to a result of [1], we are able to show that the latter has the tightest continuous relaxation bound in the family of convex reformulations of the initial non-linear model. We link both MILP and establish their hierarchy. In our models, it is not hard to observe that in an optimal solution every client will purchase automatically if able to do so. This allows us to define dominating polytopes in the respective variable spaces over which we can maximise the same objective functions to obtain an equivalent optimal solution. We will show that in that case the convex hull of mixed-integer solutions has full dimension and determine the facet-defining constraints.

In an interest to strengthen the above formulations obtained in the Master's thesis [2], we develop an adapted RLT-procedure to extract additional information from the initial non-linear model. We obtain a final formulation which is a compact extension of the previously strongest model.

Finally, we support our theoretical results with computational experiments comparing the quality of all formulations and conclude on future research to be carried out on exact methods for the SMBPP.

Keywords: *Product Pricing, Mixed-integer linear programming, Bundles*

References

[1] Billionnet, A., and S. Elloumi, and A. Lambert , *Exact quadratic convex reformulations of mixed-integer quadratically constrained problems*, Mathematical Programming. **158(1-2)** (2016), 235–266.

[2] Plein, F., and M. Labbé, *Analysis of a problem in product pricing*, Master's thesis. Department of Mathematics, Université Libre de Bruxelles, 2017.

The Rank Pricing Problem

Concepción Domínguez*
Inria, Lille, concepcion.dominguez@um.es

Herminia Calvete
Departamento de Métodos Estadísticos, Universidad de Zaragoza, herminia@unizar.es

Carmen Galé
Departamento de Métodos Estadísticos, Universidad de Zaragoza, cgale@unizar.es

Martine Labbé
Computer Science Department, Université libre de Bruxelles, mlabbe@ulb.ac.be

Alfredo Marín
Departamento de E. e Investigación Operativa, Universidad de Murcia, amarin@um.es

Abstract Pricing optimization problems aim at determining the prices of a series of products in order to maximize the revenue of a company. Setting a low price can lead to a loss of income if clients were willing to pay a higher price, but it can also make the product available to a greater amount of customers; on the contrary, a high price can generate greater revenue, but customers may not purchase it if it is too high. Therefore, a pricing problem is a bilevel program, in other words, has a hierarchical structure with a first optimization problem given by the company, which aims at maximizing its profit, and a subset of the constraints that force the solution to be optimal to another optimization problem, which is minimizing the customers' purchasing cost.

Unit-demand customers are customers interested in several products which intend to purchase at most one of them according to a selection rule. These customers were first introduced by [1]. In our problem, their purchase decision is modelled by means of a budget, since they will only buy a product provided that they can afford it. The Rank Pricing Problem arises when the selection rule of the customers depends on their preferences, that is, when customers rank the items and purchase the highest-ranked one which fits their budget. This rank-buying objective was introduced by Rusmevichientong et al. in [2], where they show that the problem is NP-complete in the strong sense and introduce a heuristic approximation algorithm together with posterior performance bounds.

The Rank Pricing Problem is introduced as a bilevel problem, and an analysis of its properties leads us to develop two mixed-integer single level formulations to tackle it. Valid inequalities are proposed taking advantage of the fact that a subset of its constraints constitutes a special case of the Set Packing Problem, and others are presented as a result of an analysis of its structure. A computational study is also included.

Keywords: *Bilevel Programming, Rank Pricing Problem, Mixed Integer Programming*

References

[1] Guruswami, V., J. D. Hartline, A. R. Karlin, D. Kempe, C. Kenyon, and F. McSherry, *On profit-maximizing envy-free pricing*, Proceedings of the sixteenth annual ACM-SIAM symposium on discrete algorithms, (2005), 1164-1173.

[2] Rusmevichientong, P., B. Van Roy, and P. W. Glynn, *A nonparametric approach to multiproduct pricing*, Operations Research, 54 **1** (2006), 82-98.

Two-level approach to solve the choice network revenue management problem

Thibault Barbier*
Department of industrial and mathematical engineering, Polytechnique Montreal,
thibault.barbier@gmail.com

Miguel Anjos
Department of industrial and mathematical engineering, Polytechnique Montreal,

Fabien Cirinei
Expretio technologies,

Gilles Savard
IVADO

Abstract Revenue management aims to sell the right product to the right customer at the right time and for the right price as explained in [4]. The overall problem is too complex and has been split into sub problems.

Generally, the first sub problem is to forecast the customers'demand. The idea is to benefit from data or/and knowledge to best suit an established choice behavior model. Independent, multinomial logit and preference list are the most common demand behavior model. This forecast accuracy is crucial because it is used by many other sub problems such as pricing, network planning or scheduling.

Among all the sub problems of revenue management, we focus on the inventory optimization sub problem. Its role is to return a control of products availability during the reservation period in order to maximize the revenue of the company. The latest forecast demand allows us to formulate this problem as an exact dynamic program [2]. However, it is too rapidly not tractable because of the number of states generated. This is why many approximations have been proposed since then and are almost always also forecast based. In order to tackle the complexity, they are often dedicated to a specific choice behavior model and exploit methods such as decomposition [3] and column generation [1].

In this presentation, we propose a new approximation for the inventory problem called Buying Tree Program (BTP). It has the advantage to be independent from the choice behavior model and is very effective thanks to a bi-level approach. Our approximation does not depend on any choice behavior because it is built on an enumeration, resulting in a tree, of any choice behavior buying paths. Rather than working with the combinatorial set of products variables as usual, we use a time variable corresponding to the end of each product sales over the reservation period. This enables us to split our problem in two parts. The first one is to determine the order of products corresponding to the time of sales end also denoted as product closing. Once this hierarchy of product closing is obtained we can more easily

compute the duration of sales for each product. This two-level approach is modeled in one mixed integer linear program which is combined with heuristic to be solved much faster.

Numerical experiments show very promising results in favor of our approximation. In the three instances tested our BTP is at least equal to the expected revenues given by best approximations but is solved in much less time. In the largest instance, our approximation outperform the existing one by solving in less than two minutes whereas others necessitate hours.

Keywords: *Revenue management, Transportation, choice models*

References

[1] Bront, J. J. M., Méndez-Díaz, I., and Vulcano, G. (2009). A column generation algorithm for choice-based network revenue management. *Operations Research*, 57(3):769–784.

[2] Gallego, G., Iyengar, G., Phillips, R., and Dubey, A. (2004). Managing flexible products on a network. Technical Report TR-2004-01, Columbia University.

[3] Liu, Q. and Van Ryzin, G. (2008). On the choice-based linear programming model for network revenue management. *Manufacturing and Service Operations Management*, 10(2):288–310.

[4] Talluri, K. and Van Ryzin, G. J. (2006). *The theory and practice of revenue management*, volume 68. Springer Science and Business Media.

Decentralized Investment through Bi-level Optimization

Stefano Nasini*
Dept. of Economics and Quantitative Methods, IESEG School of Management, CNRS,
s.nasini@ieseg.fr

Francisco-López Ramos
National Laboratory for Transportation Systems and Logistics, Mexican Institute of Transportation, flopez@imt.mx

Francisco Benita
SUTD-MIT International Design Centre, Singapore University of Technology and Design.,
francisco_benita@sutd.edu.sg

Abstract Optimization approaches for investment selection and portfolio management rely on centralized decisions about both budget allocation in different investment options and portfolio composition within the different options. However, investors do not directly select the portfolio composition, but only provide guidelines and requirements about risk control and budget allocation, relying on financial intermediaries to build the portfolio of financial securities. This results in a hierarchical investor-intermediary interaction about the overall investment decisions.

In this work, a bi-level mixed-integer quadratic optimization problem is proposed for the decentralized selection of three types of investments:

- a collection $\mathcal{M}$ of real options, with $|\mathcal{M}| = m$ and decision variables $\mathbf{z} = [z_1, \ldots, z_m]^\top$;
- a collection $\mathcal{N}$ of financial options, with $|\mathcal{N}| = n$ and decision variables $\mathbf{y} = [y_1, \ldots, y_n]^\top$;
- a risk-free asset, associated with the decision variable x.

Real options (corresponding to the particular sectors in which the investor is operating) have fixed lunching costs $c_1, \ldots, c_m$ that are defined as proportions of the available budget. These costs are associated with rates of return $B_1, \ldots, B_m$ that are specified within a given period. Since $B_1, \ldots, B_m$ are random quantities, their marginal expectations are denoted as $b_1, \ldots, b_m$, and their variance-covariance matrix as $Q^{(z} \in \mathbb{R}^{m \times m}$. The latter comprises the $q_{ij}^{(z}$ covariances between B_i and B_j. In vector form, these elements are defined as $\mathbf{c} = [c_1, \ldots, c_m]^\top$, $\mathbf{B} = [B_1, \ldots, B_m]^\top$ and $\mathbf{b} = [b_1, \ldots, b_m]^\top$.

Each financial option $i \in \mathcal{N}$ gives a return R_i, which is a random quantity with expectation r_i and their variance-covariance matrix $Q^{(y} \in \mathbb{R}^{n \times n}$. The latter comprises the $q_{ij}^{(z}$ covariances between R_i and R_j. In vector form, these elements are defined as $\mathbf{R} = [R_1, \ldots, R_n]^\top$ and $\mathbf{r} = [r_1, \ldots, r_n]^\top$.

An investor (acting as the leader) aims at maximizing a utility function by selecting the optimal proportion of money invested in the risk-free asset, x, and real options, $\mathbf{z}$. The leader's problem is

$$\begin{aligned} G(\Theta) = \quad & \min_{\mathbf{y}, x, \mathbf{z}} & & J(\mathbf{y}, x, \mathbf{z}) \\ & \text{subj. to:} & & (x, \mathbf{z}) \in \Lambda \;\text{ and }\; \mathbf{y} \in \Psi_\Theta(x, \mathbf{z}) \end{aligned} \tag{1}$$

where $\Lambda = \{(x, \mathbf{z}) \in [0,1] \times \{0,1\}^m \mid x + \mathbf{c}^\top \mathbf{z} \leq 1\}$ is the set of feasible investor decisions (i.e. verifying non-negativity, integrality and budget constraint) and $\Psi_\Theta : \mathbb{R} \times \{0,1\}^m \to 2^{[0,1]^n}$ is the solution set mapping of the financial intermediary problem for fixed decisions x and $\mathbf{z}$ by the investor. The investor utility is based on the Markovich balance between expected returns and risk [4]:

$$J(\mathbf{y}, x, \mathbf{z}) = \alpha \mathbb{V}\left[\mathbf{R}^\top \mathbf{y} + \mathbf{B}^\top \mathbf{z}\right] - (1-\alpha)\mathbb{E}\left[\rho x + \mathbf{R}^\top \mathbf{y} + \mathbf{B}^\top \mathbf{z}\right]$$

$$= \alpha \left(\mathbf{z}^\top Q^{(z}\mathbf{z} + \mathbf{y}^\top Q^{(y}\mathbf{y}\right) - (1-\alpha)\left(\rho x + \mathbf{r}^\top \mathbf{y} + \mathbf{b}^\top \mathbf{z}\right).$$

Based on this centralized decision, an intermediary (acting as the follower) builds the portfolio of financial assets by maximizing its expected rate of returns subject to a specified measures of risk. These measures can be implemented with constraints on either the variance, the value-at-risk, or maximum drawdown [1][2], reflecting the presence of legal or contractual requirements, as well as a specified degree of risk aversion.

The follower's problem is

$$\begin{array}{llll} \Psi_\Theta(x, \mathbf{z}) = \underset{\mathbf{y}}{\operatorname{argmax}} & \mathbf{r}^\top \mathbf{y} & & \\ \text{subj. to:} & x + \mathbf{y}^\top \mathbf{1} + \mathbf{c}^\top \mathbf{z} & \leq 1 & \\ & \rho x + \mathbf{r}_t^\top \mathbf{y} & \leq \Theta & t = 1 \ldots T \\ & \mathbf{y} & \geq 0, & \end{array} \tag{2}$$

where $\mathbf{r}_t = [r_{1t}, \ldots, r_{nt}]^\top$ is a realization of financial returns at period t and $\mathbf{1}$ denotes the vector of all 1's of appropriate dimension. For simplicity, by the vector inequality $\mathbf{y} \geq 0$, we mean that the componentwise inequalities $y_i \geq 0$ holds for each $i \in \mathcal{N}$.

In (2), the intermediary maximizes the benefits from the portfolio by selecting the amount of money to invest in the risky assets depending on the budget (first constraint) and the maximal drawdown limitation (second set of constraints). The latter is a linearly computable risk control [3], which mirrors the decline from a historical peak in returns (i.e. the maximal drawdown up to time t is the maximum *pain* period experienced by an investor between a peak and subsequent valley over the history of prices), specifying both the time period for the calculation and a lower bound for returns.

Single-level reformulation techniques have been applied to solve (1)-(2). In particular, we analyze the computational impact of using a two alternative reformulation approaches:

- a *KKT-based reformulation*, which characterizes the intermediary optimality based on a collection of complementary constraints;

- a *strong-duality-based reformulation*, which characterizes the intermediary optimality based on a unique prima-dual equality constraint.

Despite the equivalence of both characterization in terms of the intermediary optimality, we show that the linearization of the bi-linear terms in both reformulations give rise to substantial variations in the computational effort. In particular, the linearization of the KKT-based reformulation required the inclusion of a much larger number of binary variables and combinatorial constraints.

For both cases, the use of a collection of valid-inequalities (derived from the constraint structure of (1)-(2)) allowed speeding-up their resolution procedure, when large scale instances are taken into account. We show that the inclusion of such inequalities are capable of reducing the CPU-time up to 70% for both model reformulations.

To assess the practical consequences of the decentralized portfolio selection, we conducted computational experiments on large historical stock market data from the Center for Research in Security Prices. This allow validating and comparing the the proposed bi-level investment framework (and the resulting single-level reformulations), under different levels of investor's and intermediary's risk aversion and control. The empirical tests revel the impact of decentralization on the investment performance, and provide a comparative analysis of the computational effort corresponding to the proposed solution approaches.

To the best of our knowledge, the proposed modeling strategy represents the first mathematical programming contribution where financial intermediation is embedded into a classical investment problem. This opens new possibilities for solving real-world large scale instances of decentralized investment problems, even beyond the range of model specifications and empirical applications considered in this paper. Specifically, the following lines for further investigation can be taken into account:

- A decentralized global investment in multi-period settings where investment decisions are dynamically taken.

- A decentralized global investment in multi-market settings where a variety of intermediaries operating in different markets can be selected.

- Incorporation of further linear and quadratic programming commutable risk constraints at the intermediary's level, as well as higher moments specification (e.g. tail events) of investor's risk aversion.

- Algorithmic improvements based on cutting plane approaches, as well as specialized interior-point methods.

Keywords: *Portfolio management, Financial intermediation, Bi-level optimization*

References

[1] Mansini, R., Ogryczak, W., Speranza, M., *Conditional value at risk and related linear programming models for portfolio optimization*, Annals of Operations Research **152** (2007), 227–256.

[2] Fabian, I., *Handling CVaR objectives and constraints in two-stage stochastic models*, European Journal of Operational Research **191** (2008), 888–911.

[3] Gordon, J., Alexandre, M., *Portfolio selection with a drawdown constraint*, Journal of Banking and Finance **30** (2006), 3171 - 3189.

[4] Markowitz, H., *Portfolio selection*, The Journal of Finance **7** (1952), 77–91.

A novel Bilevel Portfolio Selection Problem

Marina Leal*
Department of Statistics and Operations Research, University of Seville, mleal3@us.es
Diego Ponce
Department of Statistics and Operations Research, University of Seville, dponce@us.es
Justo Puerto
Department of Statistics and Operations Research, University of Seville, puerto@us.es

Abstract Among the most classical research problems in operations research and financial theory we find portfolio optimization problems. Portfolio optimization is the process of choosing the proportions of various assets to be held in a portfolio, in such a way as to make the portfolio better than any other according to some criterion. Usually, the criterion combines, directly or indirectly, considerations of the expected value of the portfolio's rate of return as well as of the return's dispersion and possibly other measures of financial risk.

The classical model in this research area was originally proposed by Markowitz in 1952 [4]. This model has served as the basis for the progress of the modern portfolio financial theory. Markowitz proposed as a measure of risk the standard deviation or variance, and as a measure of the expected profit, the classical expected return. Whereas the original Markowitz model is a quadratic programming problem, many efforts have been made to linearize the portfolio optimization problem. Since Markowitz proposed his model, several other risk measures have been considered, specially those that give rise to linear programming (LP) problems (MAD, CVaR, etc.) [1].

Over time, portfolio optimization problems have become more realistic, incorporating real life aspects, among them we highlight the transaction costs. Ignoring these practical issues may result in inefficient portfolios [2], [3]. These transaction costs are the costs incurred by the investors when buying and selling assets on the markets, that are charged by the brokers or the financial institutions playing the role of intermediary. In the existing portfolio optimization problems with transactions costs, these are assumed to be fixed or variable (dependent on the amount invested in each security), but given. Nevertheless, these financial institutions paying the role of the intermediary may also fix the transaction prices trying to maximize its own profit, that is, they can also be considered optimizers in the problem.

We present a novel bilevel leader-follower portfolio selection model in which the bank has to decide on the transaction costs for investing in some securities, maximizing its benefits, and the investor has to choose his portfolio, minimizing the risk and ensuring a given expected profit. In order to minimize the risk of the investor different risk measures can be considered. This gives rise to general non linear bilevel problems in both levels. We model different bilevel versions of the problem (cooperative model, bank-leader model, ...), determine some of their properties, provide Mixed Integer Linear Programming formulations and Benders algorithms for some cases and report some computational results.

Keywords: *Portfolio Optimization, Transaction Costs, Bilevel Optimization*

References

[1] Mansini, R., Ogryczak, W. and Speranza, M.G., *On LP Solvable Models for Portfolio Selection*, Informatica. **14** (2003), 37–62.

[2] Mansini, R., Ogryczak, W. and Speranza, M.G., *Twenty years of linear programming based on portfolio optimization*, European Journal of Operational Research. **234** (2014), Issue 2, 518–535.

[3] Mansini, R., Ogryczak, W. and Speranza, M.G., *Portfolio Optimization and Transaction Costs*, in Quantitative Financial Risk Management: Theory and Practice (eds C. Zopounidis and E. Galariotis), John Wiley and Sons, Inc, Hoboken, NJ, USA. (2015).

[4] Markowitz, H.M., *Portfolio selection*, Journal of Finance. **234** (1952), 77–91.

A bilevel programming model for a problem of market regulation: application to the petrochemical industry

Héctor-Saib Maravillo-Gómez*
Facultad de Ciencias Físico-Matemáticas, Universidad Autónoma de Nuevo León,
hector.maravillogm@uanl.edu.mx

José-Fernando Camacho-Vallejo
Facultad de Ciencias Físico-Matemáticas, Universidad Autónoma de Nuevo León,
jose.camachovl@uanl.edu.mx

Martine Labbé
Département d'Informatique, Université Libre de Bruxelles, mlabbe@ulb.ac.be

Abstract In this paper, a bilevel programming model is proposed to study a problem of market regulation by the Government applied to the petrochemical industry. The problem is characterized by the state monopoly on the commercialization of inputs and the hierarchical competition between the Government and private companies for the production of final commodities. Under these conditions, at the upper level, the Government may limit the output of private companies and optimize their production of final commodities to minimize the difference between the supply and the demand for each product. On the other hand, at the lower level, the maximization of the profit for private companies is aimed. For doing this, there are two possibilities: (i) when there is a regulatory organism in private companies that distribute the inputs between them, and (ii) when each company is independent from the others. In the latter case, it is necessary to found the equilibrium between private companies. An algorithm that is based on the exploration of vertices of the lower level's dual problem is proposed to solve the proposed problem. Also, a procedure to find a set of optimal solutions is presented. Numerical experimentation is carried out using instances inspired in real life, in particular, related with the petrochemical industry.

Keywords: *Bilevel Programming, Market Regulation, Mixed Market, Petrochemical Industry*

1 Introduction

Since the period known as "Stabilizer development" and until few years, in Mexico the Constitution divided the petrochemical industry into two branches: basic petrochemical and secondary petrochemical. The economic activities that transform the natural gas and oil in raw material such as methane, ethane or naphtha belong to the basic petrochemical; and those activities that used these supplies to make oil derivates for other industries belong to the secondary petrochemical. By law, the basic petrochemical is monopolized by the Mexican government, whereas the secondary petrochemical is open to the market [1].

2 Problem Statement

The industrial structure described above has occurred in other countries and in other industries, according to specific historical, economic and political terms. The problem without considering the national and historical particularities, is the following: the Government monopolizes the production and commercialization of the raw material of an industry, in which the Government and the private companies compete in the manufacturing of the commodities. This situation implies a hierarchy of decision between the Government and the private companies. To deal with the problem, we propose a bi-level programming model, in which the Government performs as the leader and the private companies as the follower. The Government decides first how much raw material will use to manufacture commodities, thus limiting the manufacture of private companies. Besides, the Government and private companies determine the production of each commodity.

A similar situation that has been studied in the literature is called as a mixed oligopoly, which in its most classical case represents a market where at least one public company cohabits with one or more private firms[2]. Even considering a hierarchical relationship in decision making, in the case of the models that use the Stackelberg games [3]. Our model differs from that topic, because of the type of relationship between two industrial branches and the monopoly of the Government of one branch, as well as in some economic assumptions and the aim of the Government.

According to the economic theory, it is valid to assume that companies aim to maximize their profits. The economic regulation tries to solve some problems related to market failures when the maximization of individual profits does not translate into the maximization of social welfare. One of these issues occurs when the supply exceeds the demand for a commodity, which is a waste of resources; or when production is insufficient. For this reason, one of the contributions of our model is bringing in that the objective of the Government is to minimize the difference between the supply and demand, for each final commodity considered.

In some cases, the Government participation in the production yields some regulations that limit competition in the industry. Therefore, it is necessary to introduce two extreme cases. In the first case, which is the one we focus in, there is an organization that distributes the raw material among private companies, with the objective of maximizing the net profit of all private companies. In the second case, which we only analyze it analytically, there is no option and it is necessary to achieve a balance between private companies for the purchase of the raw material.

3 Proposed algorithm

To solve the resulting linear-linear bi-level programming model with continuous variables, we propose an algorithm based on the dual problem associated with the lower level. We use a one-level relaxation of the original problem, which consists in omit the objective function of the lower level. Since the dual problem of the lower level is bounded, it is possible to use the equality between the optimal value of the primal and dual problem of the lower level and to restrict these conditions to all the vertices of the polyhedron defined by the dual constraints. Iteratively, the algorithm explores the vertices of the polyhedron, thus improving the values of the objective function of the dual problem, until a stop criterion is met.

Owing to the characteristics of the problem is expected to find multiple optima. Once a sufficiently good solution is found, by the previous algorithm, the original bi-level model is reformulated as a linear programming model, which preserves the restrictions of both levels. The objective function of the upper level is replaced with the objective function of the lower level, and an equality of the objective function of the upper level with the value of the "sufficiently good" solution found previously is added. By doing this, solutions to the bi-level problem are obtained equally "good" within the inducible region.

4 Current Work

Numerical experimentation is being conducted using instances inspired in a real Mexican case. Some of the data are generated in a pseudo-random manner, but corresponding to the real oil industry. Using the procedure for obtaining multiple optimal solutions, we intend to give to the decision maker a wide range of solutions for selecting the one that is more suitable in that particular moment.

References

[1] Martínez, N., *Evolución y expresivon territorial de la industria petroquímica en México*, Invetigaciones geográficas. **46** (2001), 98-116.

[2] De Fraja, G., and Delbono, F. *Game theoretic models of mixed oligopoly*, Journal of Economic Surveys. **4** (2003), 1-17.

[3] Matsumura, T., *Stackelberg mixed duopoly with a foreign competitor*, Bulletin of Economic Researches. **55** (2003), 275-286.

Heuristics to Solve Toll Optimization Problems with Quadratic Costs

Vyacheslav Kalashnikov[1,2,3], José G. Florez-Muñiz[4], Nataliya Kalashnykova[4]

[1]*Tecnológico de Monterrey (ITESM), Campus Monterrey, Monterrey, NL, Mexico, 64849; kalash@itesm.mx*

[2]*Central Economics and Mathematics Institute (CEMI), Moscow, Russian Federation, 117418; slavkamx@mail.ru*

[3]*Sumy State University, Sumy, Ukraine, 40007; slavkmax@gmail.com*

[4]*Universidad Autónoma de Nuevo León (UANL), San Nicolás de los Garza, NL, Mexico; nkalash2009@gmail.com, jose_guadalupe64@hotmail.com*

Abstract

We consider a bilevel programming problem modeling the optimal toll assignment as applied to an abstract network of toll and free highways. A public governor or a private lease company run the toll roads and make decisions at the upper level when assigning the tolls with the aim of maximizing their profits. The lower level decision makers (highway users), however, search an equilibrium among them while trying to distribute their transportation flows along the routes that would minimize their total travel costs subject to the satisfied demand for their goods/passengers. Our model extends the previous ones by adding quadratic terms to the lower level costs thus reflecting the mutual traffic congestion on the roads. Moreover, as a new feature, the lower level quadratic costs aren't separable anymore, i.e., they are functions of the total flow along the arc (highway). In order to solve the bilevel programming problem, a heuristic algorithm making use of the sensitivity analysis techniques for quadratic programs is developed. As a remedy against being stuck at a local maximum of the upper-level objective function, we modify the well-known "filled function" method which brings us to a vicinity of another local maximum point. A series of numerical experiments conducted on test models of small and medium size shows that the new algorithm is competitive enough.

The previous century model of locating manufacturing enterprises as near as possible to the potential consumers has given way to others mainly due to the impressively rapid development of the modern transportation tools. Nowadays, producers aren't restricted too much by the large distance from the markets and can compete mainly in the areas of technologies thus enhancing the sales volumes.

During the first steps of the industrial revolution, the producing plants and factories used to be located not too far from the markets, since the goods movement was rather costly, taking a lot of time, and lacking security. With the development of the modern transportation systems, the producers are able to compete in front of the distant consumers as well, which promotes economies of scale by increasing sales volume.

However, due to the newly appearing bothers about ecology, safe transportation of hazardous materials, the complexity of the modern distribution and supply chains, the logistics costs has grown astronomically. According to the recent IMF (International Monetary Fund) estimates, logistics expenditures are responsible on average for 12% of the gross national product (GNP), while scaling 5% to 30% of the total costs at the enterprise level.

Currently, in many countries, both governmental bodies and private companies participate in gross efforts aimed at the enlargement and improvement of the transportation network's infrastructure and facilities in order to achieve the higher grade of involvement in the global economy. There are bodies engaged in the enhancement of transference and movement facilities, investment in new technologies bringing about better reliability and durability of highways and other transportation infrastructure objects. For instance, in Mexico, it is common that non-governmental (leasing) companies, non-federal (state) structures, as well as various financial institutions (like banks, holdings, etc.), are contracted with the aim of picking up the toll payments from the highway users.

The latter development taken into account, many companies and states have appended gross relevancy to the enlargement and modernization of the infrastructure to reach a better participation in the global economy. There exist bodies dealing with the improvement of communications and transference infrastructure, inventing new technologies in order to enhance the amends, excellence, and durability of the infrastructure. In Mexico, operations with new (private) highways are usually granted to non-government enterprises, state governments, or financial institutions (banks, holdings, etc.), who assign the movement toll to pick up payments from the motorists.

In the last ten years, it has been noticed that there is less heavy traffic under the concession model applied to govern these tolled highways. One of the efficient ways to elevate the use of toll highways is the tight control of the tolls (pass rates). Therefore, a natural question arises about the appropriate criteria to evaluate these rates.

Since recently, it has become clear that certain flexibility in the assignment of tolls on the crucial highways attracts more vehicles to use them and thus relaxing the heavy traffic along the free roads. Hence, a natural question arises how to evaluate the appropriate tolls for each of the toll thoroughfares. In other words, the toll optimization problem (TOP) is a crucial element of an efficient management of the transportation infrastructure.

Here, we consider the TOP as the problem of assigning optimal tolls to the arcs of a multi-commodity transportation network. The latter is usually stated as a bilevel mathematical program, in which the upper level is controlled by a leasing company (or a public administrator) who raises profits from the tolls assigned to (some) arcs of the network, while the lower level deals with an array of drivers (transportation companies) riding the cheapest paths. The problem then reads as follows: Find equilibrium among the toll values that provide high revenues being yet attractive enough to the users.

The problem in question has been examined by many prominent researchers (see the references listed below). It suffices to mention only a few high-level authors who have dealt with the TOP. Indeed, Magnanti and Wong (1984) provided a comprehensive theoretical base for the decision makers both at the upper and lower levels of the problem making use of the integer programming techniques.

Marcotte (1986) noticed that the network design problem (NDP) mainly deals with the optimal balance either of the transportation, investment, or maintenance costs of the networks subject to congestion. He also supposed that the NDP could be modeled as a multi-level mathematical program.

Dempe and Starostina (2009) contributed to the solution of TOP by designing “fuzzy” algorithms. A bit earlier, Lohse and Dempe (2005) studied TOP based on the analysis of an optimization problem that is a kind of reverse to TOP. At the same time, Didi-Biha et al. (2006) developed an algorithm for calculation of lower and upper bounds in order to determine the maximum gain from the tolls on a subset of arcs of a network transporting various commodities.

The bilevel programming offers a convenient framework modeling the toll optimization problem as it allows one to make use of the user’ behavior explicitly. In contrast to the previous works mentioned above, Labbé et al. (2000) handle TOP as a sequential game involving the owners of the highway network (the leaders) and the users (the followers) as the players, which follows exactly the structure of a bilevel program. Such a structure has also been examined by Brotcorne (1998) for the problem of fixing tariffs on load trucks running the highways. In the latter case, the leader is played by a group of competing companies, and their revenues are formed by the gross profits from the tolls, while the follower is a carrier who seeks to lower its travel expenditures, given the toll values dictated by the leader(s).

A simple TOP was studied in Kalashnikov et al. (2011), where a motorway administrator (the leader) decides the tolls on a subset of arcs of the network, whereas the users (followers) seek the shortest paths (in generalized time units) connecting the origin and destination nodes for their goods. The aim of the leader in this setting is to maximize the toll revenue. The problem could be formulated as a combinatorial program comprising NP-hard tasks, such as the Traveling Salesman Problem (*see*, Labbé et al. (1998), for a reduction method). By means of the already known NP-hardness proofs, Roch et al. (2005) obtained new results concerning the computational complexity of some existing algorithms.

Recently, Brotcorne et al. (2011) examined this problem under a bit different assumptions; namely, they permitted the network to be subsided; this is, they allowed the toll values to be unconstrained. The authors continued their work over the same problem later in Brotcorne et al. (2012) having brought up a tabu search algorithm, which helped them to infer that their heuristics did obtain better results than other combinatorial methods. Dempe and Zemkoho (2012) also explored the TOP and introduced its restatement founded on the optimal-value-function technique. This restatement is better than that making use of the Karush-Kuhn-Tucker (KKT) optimality conditions since the former accumulates the information about the congestion in the network. The authors deduced the optimality conditions appropriate for this restatement and studied some related theoretical properties of the latter.

In the majority of the above-mentioned works, the TOP in question had *linear* lower level problems. The aim of the present paper is to develop algorithms making use of the *allowable ranges to stay optimal* (ARSO) and *allowable ranges to stay basic* (ARSB) deduced with the aid of sensitivity analysis applied to the lower level quadratic problem; *cf.*, Boot (1963), Jansen (1997), Hadigheh et al. (2007). This efficient tool helps determine allowable variations of the coefficients of the objective function that do not ruin the optimality of a solution. Also, it makes one able to trace the variations in the optimal solution whenever the parameters get values beyond the ARSO or ARSB. This work has been motivated by the previous attempts described in Roch et al. (2005).

Apart from making use of the allowable ranges, the proposed algorithm also exploits the techniques of the "filled functions"; *cf.*, Renpu (1990), Wu et al. (2007), Wan et al. (2012). The latter is quite efficient when a local maximum has been run into. In that case, the "filled function" procedure allows us either to jump into a neighborhood of another local maximum, which can happen to be better or otherwise to conclude with the high probability that the best feasible optimal solution has been found. The stopping point is selected based upon certain tolerance criterion.

The validity, robustness, and the efficiency of the proposed heuristics are confirmed by the results of numerical experiments with test examples used to compare the developed approach against the other well-known algorithms.

The paper is arranged as follows: Section 2 provides the statement of the model together with the involved parameters. Namely, TOP can be analyzed as a leader-follower game that takes place on a multi-commodity network $G=(K,N,A)$ defined by a set of origin-destination couples K, a set of nodes N, and a set of arcs A. The latter is partitioned into the subset A_1 of toll arcs and the complementary subset A_2 of toll-free arcs; here $|A_1|=M_1$ and $|A_2|=M_2$, thus yielding $M=M_1+M_2$.

We endow each arc $a \in A$ with a fixed travel delay c_a and a congestion parameter d_a. Also, there is an upper limit capacity q_a associated with each arc $a \in A$ in the network. Toll arcs $a \in A_1$ also involve a toll component t_a to be determined, which, for the sake of consistency, is also expressed in time units. Letting $\left\{x_a^k\right\}_{a \in A}$ denote the set of commodity flows, and i^+ (respectively, i^-) the set of arcs having i as their head node (respectively, tail node), TOP can be formulated as a bilevel program:

TOP:

$$F(x,t) = \sum_{k \in K} \sum_{a \in A_1} t_a x_a^k \to \max_{t,x}, \tag{2}$$

subject to

$$t_a \le t_a^{max}, \qquad \forall a \in A_1, \tag{3}$$

$$t_a \ge 0, \qquad \forall a \in A_1, \tag{4}$$

$$\forall k \in K \begin{cases} x^k \in Arg \min\limits_{\bar{x}} \left[\sum\limits_{a \in A_1} (c_a + t_a) \bar{x}_a^k + \sum\limits_{a \in A_2} c_a \bar{x}_a^k + \frac{1}{2} \sum\limits_{a \in A} d_a \left(\bar{x}_a^k\right)^2 \right], \\ \text{subject to} \\ \qquad -\sum\limits_{a \in i^-} \bar{x}_a^k + \sum\limits_{a \in i^+} \bar{x}_a^k = b_i^k, \quad \forall i \in N, \\ \qquad \bar{x}_a^k \ge 0, \qquad \forall a \in A, \\ \qquad \sum\limits_{\ell \in K} \bar{x}_a^\ell \le q_a, \qquad \forall a \in A. \end{cases} \tag{5}$$

Section 3 presents the reformulation of the toll optimization problem (TOP) as a linear-quadratic bilevel programming problem. In a bit more detail: we blend the main structure of the method described in Kalashnikov et al. (2011) with a following new idea. First, we determine the vector of "fastest increase" for the upper level objective function restricted to the toll variables. The "formal gradient" of this objective F function defined in (2) can be characterized by the present flows assigned to the toll arcs:

$$\frac{\partial F}{\partial t_a}(x,t) = \sum_{k \in K} \left(x_a^k + t_a \frac{\partial x_a^k}{\partial t_a} \right), \quad a \in A_1. \tag{6}$$

According to Kalashnikov et al. (2016) and Dempe et al. (2015), the lower level Nash equilibrium problem (5) can be equivalently replaced with the quadratic programming problem with the objective function equal to the sum of all followers' objective functions:

$$f(x,t) = \sum_{k \in K} \left\{ \sum_{a \in A_1} t_a x_a^k + \sum_{a \in A} \left[c_a x_a^k + \frac{1}{2} d_a \left(x_a^k\right)^2 \right] \right\} \to \min_x, \tag{7}$$

s.t.

$$-\sum_{a \in i^-} x_a^k + \sum_{a \in i^+} x_a^k = b_i^k, \quad \forall i \in N, \forall k \in K; \tag{8}$$

$$\sum_{k \in K} x_a^k \le q_a, \quad \forall a \in A; \tag{9}$$

$$x_a^k \ge 0, \quad \forall k \in K, \forall a \in A. \tag{10}$$

Section 4 presents the theoretical background of the proposed algorithms while Section 5 deals with the algorithms' description. In particular, the algorithm's flow chart can be depicted as follows:

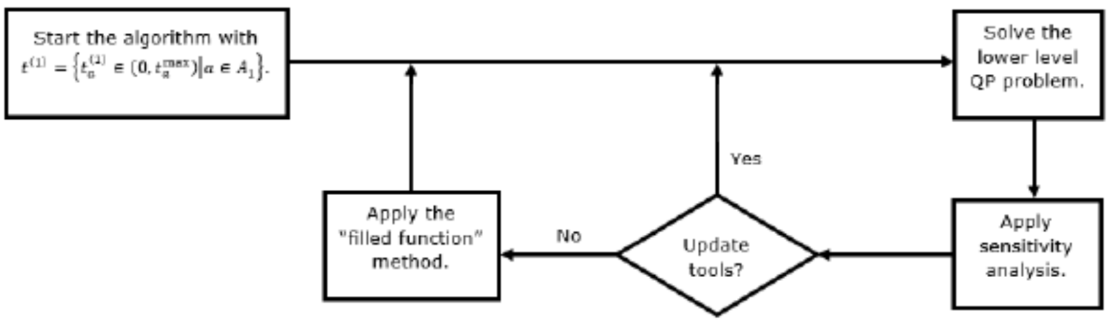


Fig. 1. The algorithm's flow chart

Figure 1 shows that first, the initial zero values assigned to the tolls. After solving the quadratic programming problem of the follower finding the flow in the arcs and yielding a value for the leader's objective function, apply sensitivity analysis taking into account only toll-arc variables. Having listed the possible increases and decreases in the coefficients of the objective function and based on the formal "gradient" vector of the upper level objective function F we update active toll vector $\{t_a^{(m)}\}_{a \in A_1}$.

When positive increments of t cannot be obtained, perform the "filled function" method. Once there is a new toll vector, go to Step 1 and close the loop. The algorithm stops when the "filled function" method fails to provide a better value for the leader's objective function after several attempts in a row, which can mean that an approximate global optimum has been reached. The multi-commodity flow corresponding to the final toll values provides the optimal solution for the followers, too.

In Section 6, the results of numerical experiments with several toll optimization test problems are presented. To apply the sensitivity analysis algorithm and the method of the "filled function" (FF), a PC Hewlett-Packard was used. The characteristics of the computer equipment used for the development and implementation of the algorithm are Intel (R) Atom (TM) CPU N455 with a speed 2.00 GHz of 1.67 GB RAM memory. The coding algorithm was written in the MATLAB mathematical software in its version MATLAB R2017a. This software was used due to its linear programming tools found in the "Optimization Toolbox". One of the functions used was *quadprog* because, in the case of unbounded capacity, i.e., $q_a = +\infty$ for all $a \in A$ in constraints (9), the lower level of the TOP can be separated without loss of generality into independent minimal cost flow problems, which are quadratic programming problems.

Section 7 comprises conclusions and targets for future research.

References

Boot, J.C.G. (1963), On sensitivity analysis in convex quadratic programming problems, *Oper. Res.* **11**: 771–786.

Brotcorne, L. (1998), *Operational and Strategic Spproaches to Traffic Routers' Problems*, Ph.D. dissertation, Université Libre de Bruxelles (*in French*).

Brotcorne, L., Cirinei, F., Marcotte, P., and Savard, G. (2011), An exact algorithm for the network pricing problem, *Discrete Optim.* **8:** 246–258.

Brotcorne, L., Cirinei, F., Marcotte, P., and Savard, G. (2012), A tabu search algorithm for the network pricing problem, *Comput. Oper. Res.* **39:** 2603–2611.

Dempe, S., and Starostina, T. (2009), Optimal toll charges: A fuzzy optimization approach. - In: F. Heyde, A. Lóhne, C. Tammer (Eds.), *Methods of Multicriteria Decision – Theory and Applications,* Aachen: Shaker Verlag, pp. 29–45.

Dempe, S., and Zemkoho, A.B. (2012), Bilevel road pricing: Theoretical analysis and optimality conditions, *Ann. Oper. Res*. **196**: 223–240.

Dempe, S., Kalashnikov, V.V., Pérez-Valdés, G.A., and Kalashnykova, N.I. (2015), *Bilevel Programming Problems. Theory, Algorithms, and Applications to Energy Networks*. Heidelberg-New York-Dordrecht-London: Springer.

Didi-Biha, M., Marcotte, P., and Savard, G. (2006), Path-based formulation of a bilevel toll setting problem. – In: S. Dempe and V.V. Kalashnikov (Eds.), *Optimization with Multi-Valued Mappings: Theory, Applications, and Algorithms*, Boston: Springer Science, pp. 29–50.

Hadigheh, A.G., Romanko, O., and Terlaky, T. (2007), Sensitivity analysis in convex quadratic optimization: Simultaneous perturbation of the objective and right-hand-side vectors, *Algorithmic Oper. Res*. **2**: 94–111.

Jansen, B. (1997), *Interior Point Techniques in Optimization. Complementarity, Sensitivity, and Algorithms*. Dordrecht, the Netherlands: Springer-Science+Business Media, B.V.

Kalashnikov, V.V., Kalashnykova, N.I., and Herrera-Maldonado, R.C. (2011), Solving bilevel toll optimization problems by a direct algorithm using sensitivity analysis. – In: R.C. Clute (Ed.), *Proc. 2011 New Orleans Int. Acad. Conf.*, New Orleans, LA, March 21-23, pp. 1009 –1018.

Kalashnikov, V.V., Herrera-Maldonado, R.C., Camacho-Vallejo, J.F., and Kalashnykova, N.I. (2016), A heuristic algorithm solving bilevel toll optimization problems, *Int. J. Logistics Management*, **27**: 31– 51.

Labbé, M., Marcotte, P., and Savard, G. (1998), A bilevel model of taxation and its applications to optimal highway pricing, *Manag. Sci*. **44**: 1608–1622.

Labbé, M., Marcotte, P., and Savard, G. (2000), On a class of bilevel programs. – In: G. Di Pillo and F. Giannessi (Eds.), *Nonlinear Optimization and Related Topics*. Dordrecht: Kluwer Academic Publishers, pp. 183–206.

Lohse, S., and Dempe, S. (2005), *Best Highway Toll Assigning Models and an Optimality Test*. Preprint, TU Bergakademie Freiberg, Nr. 2005-6, Fakultät für Mathematik und Informatik, Freiberg (*in German*).

Magnanti, T.L., and Wong, R.T. (1984), Network design and transportation planning: Models and algorithms, *Transp. Sci*. **18**: 1–55.

Marcotte, P. (1986), Network design problem with congestion effects: a case of bilevel programming, *Math. Programming*, **34**: 142–162.

Renpu, G.E. (1998), A filled function method for finding a global minimizer of a function of several variables, *Math. Programming*, **46**: 191–204.

Roch, S., Savard, G., and Marcotte, P. (2005), Design and analysis of an algorithm for Stackelberg network pricing, *Networks*, **46**: 57–67.

Wan, Z., Yuan, L., and Chen, J. (2012), A filled function method for nonlinear systems of equalities and inequalities, *Comput. App. Math*. **31**: 391–405.

Wu, Z.Y., Mammadov, M., Bai, F.S., and Yang, Y.J. (2007), A filled function method for nonlinear equations, *App. Math. Comput*., **189**: 1196–1204.

Railway Demand Forecasting: A Machine Learning Approach

Neda EtebariAlamdari*
Mathematics and Industrial Engineering , Polytechnique Montreal,
neda.etebarialamdari@polymtl.ca

Gilles Savard
Mathematics and Industrial Engineering , Polytechnique Montreal ,
gilles.savard@polymtl.ca

Miguel F. Anjos
Mathematics and Industrial Engineering , Polytechnique Montreal ,
miguel-f.anjos@polymtl.ca

Abstract

Demand forecasting in transportation is about the estimation of the number of passengers aiming to travel on a specific origin-destination within a given time window. These forecasted values are of great importance in a revenue management system. In railway industry, a precise forecasted demand could remarkably improve the seats allocation and consequently, maximize the firm's revenue [1].

Instead of using a common econometric model to modelize the demand, we introduce a comprehensive machine learning approach to simulate the behaviors of the customers. Incorporating these customers' behaviours into an optimization program leads to a bilevel program where the leader controls variables such as price and capacity, while customers react with the goal of optimizing their utility over the leader's offers.[2].

In this talk, we applied various machine learning algorithms and different preprocessing techniques to predict the future bookings of customers in railway industries for two different aggregation levels; a general high-level and a detailed and complex aggregation level.

A key step in the proposed approach is to optimally set all the hyper-parameters of the algorithms such that we can obtain the highest possible accuracy while keeping our method generalizable and stable while keeping an acceptable computing time.

Our results show that by using a greedy search in a well-defined stacked generalization method, combined with proper preprocessing and post-processing techniques, we improve the forecasts for both aggregation levels.

Keywords: *demand forecasting, bilevel programming, machine learning, optimization*

References

[1] Azadeh, Sh Sharif, Patrice Marcotte, and Gilles Savard, *A non-parametric approach to demand forecasting in revenue management*, Computers & Operations Research.**63** (2015): 23–31.

[2] Marcotte, Patrice, and Gilles Savard. *A bilevel programming approach to optimal price setting*, Decision & Control in Management Science. Springer, Boston, MA, 2002. 97-117.

Designing a road network with toll setting and hazmat transportation

Stefano Nasini*
Dept. of Economics and Quantitative Methods, IESEG School of Management, CNRS, s.nasini@ieseg.fr

Francisco-López Ramos
National Laboratory for Transportation Systems and Logistics, Mexican Institute of Transportation, flopez@imt.mx

Armando Guarnaschelli
School of Transportation Engineering, Pontificia Universidad Católica de Valparaíso, armando.guarnaschelli@pucv.cl

Abstract Network operators face problems such as traffic congestion and exposure of population to risk of hazardous materials. To alleviate these problems, investments on new roads are planned, and toll charging policies defined. In the literature, these practices are known as the Network Design Problem (NDP) [1] and the Toll Setting Problem (TSP) [2], respectively. The NDP refers to ordinary vehicles, and when hazmat vehicles are studied, instead, it is known as the Hazmat Transportation Problem (HTP) [3].

Current trends of research, only integrate either the NDP and HTP or the TSP and HTP and the NDP and TSP, but not all three together. However, it is important for both the network operator and the vehicles traveling throughout this network. On the one hand, the network operator should consider all factors in the investment on new roads, such as the flow of ordinary and hazmat vehicles. On the other hand, vehicles can choose among more alternative routes based not only on travel time but also on costs due to toll charging and traffic congestion.

Our contribution proposes an innovative mathematical programming approach to simultaneously solve the NDP, TSP, and HTP. The integrated Network Design, Toll Setting and Hazmat Transportation Problem (NTHP) can be defined as a leader-follower game that materializes on a multi-commodity network $\mathcal{G} = (\mathcal{V}, \mathcal{K}, \mathcal{N}, \mathcal{A})$.

Before presenting the mathematical programming formulation for the NTHP, the following notation is introduced:

Sets:

$\mathcal{N}$	Set of nodes.
$\mathcal{A}$	Set of links between nodes (with $\mathcal{A} \subseteq \mathcal{N} \times \mathcal{N}$)
$\mathcal{A}^N$	Set potential links between nodes (with $\mathcal{A}^N \subseteq \mathcal{A}$)
$\mathcal{K}$	Set of origin-destination pairs (with $\mathcal{K} \subseteq \mathcal{N} \times \mathcal{N}$).
$\mathcal{V}$	Set of vehicle types.

Exogenous parameters:

c_a	Cost of constructing new link $a \in A^N$.
n_a	Number of people exposed to hazmat risk on link $a \in A$.
q_a	Maximum number of vehicles going through link $a \in \mathcal{A}$.
$g^{v,k}$	Number of vehicles traveling from origin $o(k) \in \mathcal{N}$ to destination $d(k) \in \mathcal{N}$.

Operator's decision variables:

t_a^v	Unitary toll at each link $a \in \mathcal{A}$ and for each vehicles type $v \in \mathcal{V}$.
y_a	Binary decision concerning the construction of new link $a \in \mathcal{A}^N$.

Vehicles's decision variables:

$x_a^{v,k}$	Number of vehicles of type $v \in \mathcal{V}$, with trajectory $k \in \mathcal{K}$, going through $a \in \mathcal{A}$.

Set $\mathcal{K}$ is partitioned into subsets $\mathcal{K}^v$, for each $v \in \mathcal{V} : \mathcal{K}^v \neq \emptyset$, accounting for the commodities (vehicles) classified according to their type (i.e., 0-regular and 1-hazmat). Additionally, a set $\mathcal{A}$ is decomposed into toll links subset $\mathcal{A}^T$ and toll-free links subset $\mathcal{A}^F$. Beside, we introduce the vector notation $\mathbf{t} = [t_a^v \ : \ a \in \mathcal{A}, v \in \mathcal{V}] \in \mathbb{R}^{|\mathcal{A}||\mathcal{V}|}$, $\mathbf{y} = [y_a \ : \ a \in \mathcal{A}] \in \{0,1\}^{|\mathcal{A}|}$ and $\mathbf{t} = [x_a^{v,k} \ : \ a \in \mathcal{A}, v \in \mathcal{V}, k \in \mathcal{K}] \in \mathbb{R}^{|\mathcal{A}||\mathcal{V}||\mathcal{K}|}$, which will be used in the rest of the paper. Finally, the operator's and vehicles' payoffs are defined to capture their respective revenue and cost structures as follows:

$$\text{Operator's payoff:} \quad P_O\Big(\mathbf{t}, \mathbf{y}, \mathbf{x}\Big) = \sum_{a \in A^T} \sum_{v \in \mathcal{V}} t_a^v \sum_{k \in \mathcal{K}^v} x_a^{v,k} - \sum_{a \in A^N} c_a y_a - H_O \sum_{a \in A} n_a \sum_{k \in \mathcal{K}^1} x_a^{1,k}$$

$$\text{Vehicles's payoff:} \quad P_V\Big(\mathbf{t}, \mathbf{y}, \mathbf{x}\Big) = -\sum_{a \in A^T} \sum_{v \in \mathcal{V}} t_a^v \sum_{k \in \mathcal{K}^v} x_a^{v,k} - H_V \sum_{a \in A} u_a(x_a^{v,k} \ : \ v \in \mathcal{V}, k \in \mathcal{K})$$

The operator's payoff is the total profit consisting of the revenue from toll value (t_a^v, for $a \in \mathcal{A}$, $v \in \mathcal{V}$) multiplied by commodity flows ($x_a^{v,k}$, for $a \in \mathcal{A}$, $v \in \mathcal{V}$, $k \in \mathcal{K}$), minus the two sources of cost: (i) the cost associated with the construction of new links ($y_a c_a$, for $a \in A^N$) and (ii) the cost related to risk exposure to hazmat transportation. This second cost is a social cost that a benevolent operator must internalize. This is compute as n_a (the size of the population in the neighborhood of link a), and H_O renders the market value of such risk.

The vehicles payoff is the negative of its transportation cost, which consists of the toll charges established by the operator (first term) and the time cost defined as a convex function of the amount of flow (second term), $u_a(x_a^{v,k} \ : \ v \in \mathcal{V}, k \in \mathcal{K})$, where H_V is the market value of the individual cost associated with travel time.

The TNHP is formulated as the following bi-level model:

$$\max_{\mathbf{t},\mathbf{y},\mathbf{x}} \quad P_O\Big(\mathbf{t}, \mathbf{y}, \mathbf{x}\Big) \tag{1a}$$

$$\text{subject to:} \quad \mathbf{c}^\top \mathbf{y} \leq B \tag{1b}$$

$$0 \leq \mathbf{t} \leq \mathbf{T}, \quad \text{and} \quad \mathbf{y} \in \{0,1\}^{|\mathcal{A}^N|} \tag{1c}$$

$$\mathbf{x} \in \Psi(\mathbf{t}, \mathbf{y}) \tag{1d}$$

where $\mathbf{T} = [T_a^v \ : \ a \in \mathcal{A}, v \in \mathcal{V}]$ is the vector of maximum tolls; the constraint (1b) limit the number of new links that can be constructed according to the investment budget B; constraints (1c) set variables domains – i.e. non-negativity and upper bounds on toll variables, and integrality on capacity expansion.

The vehicle best response $\Psi(\mathbf{t}, \mathbf{y})$ results from the solution of the following multicommodity network flow problem:

$$
\begin{aligned}
\Psi(\mathbf{t},\mathbf{y}) = \quad \underset{\mathbf{x}}{\operatorname{argmax}} \quad & P_V\Big(\mathbf{t},\mathbf{y},[\mathbf{x}^{v,k}\ :\ v\in\mathcal{V},k\in\mathcal{K}]\Big) && && (2a)\\
\text{subject to:} \quad & N\mathbf{x}^{v,k} = \boldsymbol{w}^{v,k} && \text{for}\ \ v\in V,\ k\in K^v && (2b)\\
& r^v \sum_{k\in\mathcal{K}^v} \mathbf{x}^{v,k} \le Q[\mathbf{1};\mathbf{y}], && \text{for}\ \ v\in\mathcal{V} && (2c)\\
& \mathbf{x}^{v,k} \ge 0, && \text{for}\ \ v\in\mathcal{V},\ k\in\mathcal{K}^v && (2d)
\end{aligned}
$$

In (2b), $\mathbf{x}^{v,k} = [x_a^{v,k}\ :\ a\in\mathcal{A}] \in \mathbb{R}^{|\mathcal{A}|}$ is the vector of vehicle flow of type (v,k) circulating through the network, $N \in \{-1,0,1\}^{|\mathcal{N}|\times|\mathcal{A}|}$ is the incidence matrix, and vector $\boldsymbol{w}^{v,k} = [w_i^{v,k}\ :\ i\in\mathcal{N}]$ corresponds to the in-flow and out-flow at each node. In (2c), matrix $Q \in \mathbb{R}^{|\mathcal{A}|\times|\mathcal{A}|}$ is diagonal with elements q_a denoting the capacity of link a with respect to a established type of vehicle $v\in V$, i.e. the maximum number of vehicles of a established type v which can pass through a. Vector $[\mathbf{1};\mathbf{y}] \in \{0,1\}^{|\mathcal{A}|}$ contains $|\mathcal{A}|-|\mathcal{A}^N|$ consecutive ones and $|\mathcal{A}^N|$ binary values, corresponding to the operator's decision about the creation of new links y_a, for $a\in\mathcal{A}^N$. Finally, constant r^v normalizes the vehicle units in terms of a established vehicle type.

The MINLBP (1)–(2a) is approximated by a mixed-integer linear problem, based on the inclusion of a large collection of binary variables and constraints to linearize the convex function $u_a(x_a^{v,k}\ :\ v\in\mathcal{V},k\in\mathcal{K})$ and the bi-linear terms resulting from the single level reformulation. The accuracy of the approximation can be made arbitrarily high by allowing a more *granular* segmentation of the non-linear terms (i.e. the inclusion of a larger amount of binary variables and constraints).

Decomposition approaches are proposed to deal with the resulting large scale mixed-integer linear problem with different levels of *granularity* in the reformulation–linearization. This shows the capability of the proposed approach to provide accurate solutions (small GAP in comparison with their high granularity counterpart) in a small computational time. The proposed decomposition approaches combine the standard ILO-CPLEX implementation of the Branch & Cut with the Benders cutting-plane approach.

A case study based on an extended road network of Sioux Falls city is used for validation.

Keywords: *Road Network Design, Toll Setting, Hazmat Transportation, mixed-integer non-linear bi-level problem, approximation, single level reformulation*

References

[1] Gao, Z., Wu, J., Sun, H., *Solution algorithm for the bi-level discrete network design problem*, Transportation Research Part B **39** (2005), 479–495.

[2] Labbé, M., Marcotte, P., Savard, G., *A Bilevel Model of Taxation and Its Application to Optimal Highway Pricing*, Management Science **44** (1998), 1608–1622.

[3] Kara, B., Verter, V., *Designing a Road Network for Hazardous Materials Transportation*, Transportation Science **38** (2004), 188–196.